AdS backgrounds in five-dimensional $\mathcal{N} = 2$ supergravity

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Motivation: The AdS/CFT correspondence

Main idea: supersymmetric AdS backgrounds

AdS_5 backgrounds and their moduli spaces

Examples from type IIB supergravity

Motivation: The AdS/CFT correspondence

AdS/CFT conjecture

Consider a stack of N $D3$ branes in type IIB string theory.

- on the branes: open strings $\to$ $4d$ $SU(N)$ gauge theory
- in the bulk: closed strings $\to$ relativistic gravity on AdS_5
- idea [Maldacena '98]:

$$\mathrm{AdS} = \mathrm{CFT}$$

Example: $\mathrm{AdS}_5 \times S^5$

$\mathcal{N} = 4$, $d = 4$ SCFT $\leftrightarrow$ type IIB supergravity on $\mathrm{AdS}_5 \times S^5$

- simplest example
- relation between quantum field theory and classical SUGRA
- but: very symmetric $\Rightarrow$ restricted

Generalizations: Sasaki-Einstein geometry

- replace S^5 by a general compact Sasaki-Einstein manifold
- resulting theory preserves 8 real supercharges ($\mathcal{N} = 2$) on AdS_5
- dual to $\mathcal{N} = 1$ SCFT in 4D
- explicit family of examples: $Y^{p,q}$ manifolds [Gauntlett et. al., 06']

Explicit relations

- global symmetry of SCFT $\leftrightarrow$ gauge group of AdS background
- supersymmetric deformation space of SCFT $\leftrightarrow$ supersymmetric moduli space of AdS background
- Question: Relation between SCFT and AdS background without compactification?

Main idea: supersymmetric AdS backgrounds

Supersymmetric AdS backgrounds

- classical supersymmetric solutions of supergravity with $\Lambda < 0$
- play a key role in the AdS/CFT correspondence
- AdS moduli spaces are related to conformal manifolds of SCFTs

Idea: Study general conditions on AdS solutions preserving $\mathcal{N}$ supersymmetry in a d-dimensional supergravity.

[de Alwis, Louis, McAllister, Triendl, Westphal '14; Louis, Triendl '14; Louis, Lüst '15; Louis, Triendl, Zagermann '15; Louis, CM '16; Louis, Lüst '16; Louis, CM '16]

General method: background conditions

Start with a general supergravity theory that contains scalars Φ and fermions Ψ. Denote background values by $\langle \cdot \rangle$.

- spectrum is determined by number of supersymmetries and dimension d
- Lorentz invariance: only $\langle \Phi \rangle \neq 0$
- maximal preserved supersymmetry: $\langle \delta_\epsilon \Psi \rangle = 0$
  $\Rightarrow$ conditions $\langle F(\Phi) \rangle = 0$
- $\langle V(\Phi) \rangle < 0$ negative cosmological constant

General method: moduli spaces

- expand scalars $\Phi = \langle \Phi \rangle + \delta\Phi$
- compute conditions $\langle \delta F(\Phi) \rangle = 0$
- moduli space is given as

$$\mathcal{M} = \{\text{solutions } \delta\Phi \text{ to } \langle \delta F(\Phi) \rangle = 0\}$$

- caution: remove unphysical degrees of freedom (Goldstone bosons)

AdS_5 backgrounds and their moduli spaces

Field content

[Bergshoeff et. al, '04]

- here: 8 real supercharges ($\mathcal{N} = 2$)
- gravity multiplet

$$(g_{\mu\nu}, A^0_\mu, \psi^{\mathcal{A}}_\mu) \quad \mathcal{A} = 1, 2$$

- n_V vector multiplets

$$(A^i_\mu, \lambda^{i\mathcal{A}}, \phi^i) \quad i = 1, ..., n_V$$

- n_H hypermultiplets

$$(q^u, \zeta^\alpha) \quad u = 1, ..., 4n_H \quad \alpha = 1, ..., 2n_H$$

Scalar fields

- consider only scalar fields
- these are maps from space time M_5 to a target space

$$\phi^i \otimes q^u : M_5 \longrightarrow \mathcal{T}_V \times \mathcal{T}_H$$

- $\mathcal{T}_V$ is a projective special real manifold
- $\mathcal{T}_H$ is a quaternionic Kähler manifold

Projective special real manifolds

Projective special real manifold $(\mathcal{T}_V, g)$ is defined as follows.

- let h^I, $I = 0, ..., n_V$ be coordinates on $\mathbb{R}^{n_V+1}$
- for a cubic homogeneous polynomial $P(h^I)$, consider

$$\mathcal{T}_V = \{P(h^I) = 1\}$$

- then a positiv metric for the h^I's is given by

$$a_{IJ} = -\tfrac{1}{3}\partial_I \partial_J \log P$$

- solve $P(h^I(\phi)) = 1$ for $\phi^i \Rightarrow$ coordinates on $\mathcal{T}_V$
- endow $\mathcal{T}_V$ with the pullback metric

$$g_{ij} = \partial_i h^I \partial_j h^J a_{IJ}$$

Quaternionic Kähler manifolds

Quaternionic Kähler manifold $(\mathcal{T}_H, G, Q)$ is defined by

- a Riemannian metric G on $\mathcal{T}_H$
- a ∇^G-invariant rank three subbundle $Q \subset \mathrm{End}(T\mathcal{T}_H)$
- Q is locally spanned by a triplet J^n of almost complex structures s.t.

$$J^1 J^2 = J^3$$

- ∇^G rotates endomorphisms inside Q, i.e.

$$\nabla J^n := \nabla^G J^n - \epsilon^{npq}\theta_p J_q = 0$$

Isometries of $\mathcal{T}_H$

- isometries of $\mathcal{T}_H$ are given by triholomorphic Killing vectors k_I
- this defines associated triplets of moment maps satisfying

$$\nabla \mu_I^n = -\tfrac{1}{2} G \circ J^n(k_I)$$

- introduce "dressed" quantities

$$k := h^I k_I, \qquad \mu^n := h^I \mu_I^n$$

- gauging of the hypermultiplet scalars q^u by

$$\mathcal{D}q^u = dq^u + k_I^u A^I$$

Scalar potential

- gauging also introduces a scalar potential V, i.e. a function

$$V : \mathcal{T}_V \times \mathcal{T}_H \longrightarrow \mathbb{R}$$

- here:

$$V(\phi, q) = 2G(k, k) + 2g(\partial\mu^n, \partial\mu_n) - 4\mu^n\mu_n$$

- negative cosmological constant only from $\mu^n\mu_n$

Fermionic supersymmetry variations

- fermionic shift matrices are given in terms of isometries on $\mathcal{T}_H$
- gravitino variation

$$\delta_\epsilon \psi_\mu^{\mathcal{A}} = D_\mu \epsilon^{\mathcal{A}} - i\mu^n \sigma_n^{\mathcal{A}\mathcal{B}} \epsilon_{\mathcal{B}} + ...$$

- gaugino variation

$$\delta_\epsilon \lambda^{i\mathcal{A}} = -\partial_i \mu^n \sigma_n^{\mathcal{A}\mathcal{B}} \epsilon_{\mathcal{B}} + ...$$

- hyperino variation

$$\delta_\epsilon \zeta^\alpha = \frac{\sqrt{6}}{4} \mathcal{U}^{\alpha\mathcal{A}}(k) \epsilon_{\mathcal{A}} + ...$$

for $\mathcal{U}^{\alpha\mathcal{A}}$ an invertible vielbein on $\mathcal{T}_H$

Maximally supersymmetric AdS backgrounds I

- maximal supersymmetry $\Rightarrow$ fermionic SUSY variations vanish

$$\langle \delta_\epsilon \psi_\mu^{\mathcal{A}} \rangle = \langle \delta_\epsilon \lambda^{i\mathcal{A}} \rangle = \langle \delta_\epsilon \zeta^\alpha \rangle = 0$$

- gaugino and hyperino variation imply

$$\langle k \rangle = \langle h^I k_I \rangle = 0, \quad \langle \partial \mu^n \rangle = \langle \partial h^I \mu_I^n \rangle = 0$$

- $\Rightarrow U(1)_R$ symmetry always unbroken in the vacuum [Tachikawa '06]

Maximally supersymmetric AdS backgrounds II

- gravitino variation $\Rightarrow$ Killing spinor equation
- this implies

$$\langle \mu^n \rangle = \langle h^I \mu_I^n \rangle = \lambda v^n \quad v \in S^2$$

- $\lambda \in \mathbb{R}$ is related to the cosmological constant
- $\Rightarrow U(1)_R$ symmetry is gauged by the graviphoton $\langle h^I \rangle A_I$

Moduli space

- expand $\phi^i = \langle \phi^i \rangle + \delta\phi^i$, $q^u = \langle q^u \rangle + \delta q^u$
- require variations of background conditions to vanish

$$\langle \delta k \rangle = \langle \delta \mu^n \rangle = \langle \delta \partial \mu^n \rangle = 0$$

- one finds: variations $\delta\phi^i$ are fixed, but δq^u only constrained
- $\Rightarrow$ moduli space $\mathcal{M} \subset \mathcal{T}_H$

Structure of the moduli space I

Proposition

The space $\mathcal{M}$ of physical deformations is a Kähler manifold.

Sketch of the proof:

- remove unphysical deformations (Goldstone bosons) $\delta q^u \propto \langle k_I^u \rangle$
- use $SU(2)_R$ symmetry to rotate $\langle \mu^n \rangle = \lambda v^n$ in z-direction
- one shows: constraints on δq^u are J^3 invariant
- $\Rightarrow (\mathcal{M}, G, J^3)$ is an almost hermitian manifold

Structure of the moduli space II

- use the following [Alekseevsky, Marchiafava '00]:

Theorem

An almost hermitian submanifold (M, G, J) of a quaternionic Kähler manifold $(\tilde{M}, \tilde{G}, Q)$ is Kähler if and only if there exists a section I of Q that anticommutes with J and

$$I(T_pM) \perp T_pM \quad \forall p \in M.$$

- here: $J = J^3$ and $I = J^1$
- constraints on δq^u imply $G \circ J^1 \equiv 0$ on $\mathcal{M}$
- $\Rightarrow (\mathcal{M}, G, J^3)$ is Kähler

Examples from type IIB supergravity

Type IIB solutions with AdS factors

- let SE_5 be a five-dimensional compact Sasaki-Einstein manifold
- consider type IIB solutions of the form $\mathrm{AdS}_5 \times \mathrm{SE}_5$
- such solutions preserve 8 real supercharges
- $\Rightarrow$ AdS/CFT relates them with $\mathcal{N} = 1$ SCFTs

Question: How much of the ten-dimensional solution can we see in five-dimensional AdS backgrounds?

Consistent truncations

Idea: use consistent truncations!

- expand 10D fields into 5D fields on AdS_5 and Sasaki-Einstein structure forms
- consistency condition: 5D solutions must lift to 10D solutions
- resulting 5D theories preserve at least 8 supercharges
  $\Rightarrow$ study AdS backgrounds of truncated theories

Compactifications on $T^{1,1}$

- here: study Sasaki-Einstein manifold $T^{1,1} = \frac{SU(2)\times SU(2)}{U(1)}$
- dual to Klebanov-Witten theory
- transitiv $SU(2) \times SU(2)$ action simplifies truncations
- 10D moduli space is complex five-dimensional [Ashmore et. al, '16]

The Betti-hyper truncation

- study truncation containing 1 vector multiplet and 3 hypermultiplets
- scalar target spaces

$$\mathcal{T}_V = SO(1,1), \quad \mathcal{T}_H = \tfrac{SO(4,3)}{SO(4)\times SO(3)}$$

- we find: this truncation admits AdS backgrounds
- but only two complex moduli: Axion-Dilaton τ + geometric modulus z
- moduli space is a torus bundle

$$T^2 \hookrightarrow \mathcal{M} \longrightarrow SU(1,1)/U(1)$$

Identification of the moduli space

Quesiton: What is the total space $\mathcal{M}$?

- compute Kähler potential of $\mathcal{M}$

$$K = -4\log(\tau - \bar{\tau}) - i\lambda\kappa_5 \tfrac{(z-\bar{z})^2}{\tau-\bar{\tau}}$$

- compare with Kähler potential of $SU(2,1)/U(2)_\epsilon$ for $\epsilon \in \mathbb{R}$

$$L_\epsilon = -\log(\tau - \bar{\tau} + i\epsilon(z-\bar{z})^2)$$

- expansion for small $\epsilon = 4\lambda\kappa_5$

$$L_\epsilon \simeq \tfrac{1}{4}K + \mathcal{O}(\epsilon^2)$$

- $\Rightarrow \mathcal{M} = SU(2,1)/U(2)_{\kappa_5 \to 0}$

Summary of results

- AdS_5 gauge groups contain $U(1)_R$ factor
- moduli space is Kähler
- explicit moduli space metrics from type IIB supergravity
- but: consistent truncations do not contain all moduli

Outlook

- study AdS_5 backgrounds in terms of $U(1)_R$ actions on quaternionic Kähler manifolds
- compute metric corresponding to Betti-hyper truncation in Klebanov-Witten theory
- repeat computations in $d = 3$ supergravity
- obtain unifying formulation of AdS backgrounds in supergravity