Phenomenological aspects of magnetized SYM theories in higher dimensions

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5th String Theory in the Greater Tokyo Area
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@ Waseda U. & Tokyo Metropolitan U.

Plan of this talk

I. Motivation
II. SYM on magnetized tori
III. Phenomenological aspects
    MSSM-like models (visible sector)
    DSB models (hidden sector)
IV. Summary

MOTIVATION

What is the most fundamental element of our world?

- Classical elements in ancient Greece

  Earth, Water, Air, Fire (, Aether)

- Atoms

  H, He, Li, Be, B, C, N, O, F, Ne, …

- Proton, neutron & electron

What is the most fundamental element of our world?

- Three generations of quarks and leptons interacting with (through) gauge and Higgs bosons

Quarks Q^I, U^I, D^I $\qquad I = 1{,}2{,}3$

Leptons L^I, N^I, E^I

Gluons $G_\mu^{1,\ldots,8}$, weak bosons $W_\mu^\pm, Z_\mu$, photon A_μ

Higgs boson H

What is the most fundamental element of our world?

- The standard model of elementary particles

(Spontaneously broken) Chiral gauge theory

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

Quarks Q^I, U^I, D^I $\quad I = 1,2,3$

Leptons L^I, N^I, E^I

Gauge bosons $G_\mu^{1,\dots,8},\ A_\mu^{1,2,3}, B_\mu \to W_\mu^\pm, Z_\mu, A_\mu$

Higgs boson H

What is the most fundamental element of our world?

- The standard model of elementary particles

(Spontaneously broken) Chiral gauge theory

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

Extremely successful model up to TeV scale

– All the particle contents discovered

– Parameters are (being) measured precisely

What is the most fundamental element of our world?

- Why so many fundamental elements, those have different properties?

Quarks Q^I, U^I, D^I $\qquad I = 1,2,3$

Leptons L^I, N^I, E^I

Gauge bosons $G_\mu^{1,\dots,8},\ A_\mu^{1,2,3}, B_\mu$

Higgs boson H

What is the most fundamental element of our world?

- Why so many fundamental elements, those have different properties?

Quarks Q^I, U^I, D^I

Leptons L^I, N^I, E^I

Gauge bosons $G_\mu^{1,\dots,8}$

Higgs boson H

	Observed (GeV)
(m_u, m_c, m_t)	$(2.3 \times 10^{-3}, 1.28, 1.74 \times 10^2)$
(m_d, m_s, m_b)	$(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18)$
(m_e, m_μ, m_τ)	$(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78)$
$\lvert V_{\rm CKM} \rvert$	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$

Particle Data Group Collaboration (Beringer, J. et al.) Phys.Rev. D86 (2012) 010001

God put the hierarchy among the elements?

What is the most fundamental element of our world?

- Superstring theory may answer the question
  - Massless modes yield gauge (YM) theories

    The oscillations of a single string

    → appearance of all the observed particles?
  - The extra-dimensional space can be the source of hierarchies
  - Does SM appear at a low energy with viable values of parameters?

What is the most fundamental element of our world?

- Superstring theory may answer the question
  - The low-energy effective theory:

    Supersymmetric Yang-Mills (SYM) theory

    in various dimensions

N Dp-branes → $U(N)$ SYM in (p+1)-dim. spacetime

The simplest one: 10D SYM (consists of a single multiplet)

→ lower-dim. SYM theories

10D SYM on magnetized tori

Basic features

- Degenerated chiral zero-modes appear
- The degeneracy is determined by the number of fluxes
- 4D Yukawa couplings are calculable

D. Cremades, L.E. Ibanez & F. Marchesano, JHEP 0405 (2004) 079

Applications

- Phenomenological models for visible and hidden sectors

Further aspects

- D-brane interpretations and dual descriptions

SYM ON MAGNETIZED TORI

QED on $M_4 \times T^2$

- Lagrangian

$$\mathcal{L} = -\frac{1}{4g^2}\left(F^{MN}F_{MN} - 2i\bar{\lambda}\Gamma^M D_M\lambda\right)$$

$$D_M\lambda = (\partial_M - iA_M)\lambda$$

- Torus compactification $x_M = (x_\mu, y_m)$ $m = 4, 5$

$$y_m \sim y_m + n_m \ , \ n_m = \text{integer}$$

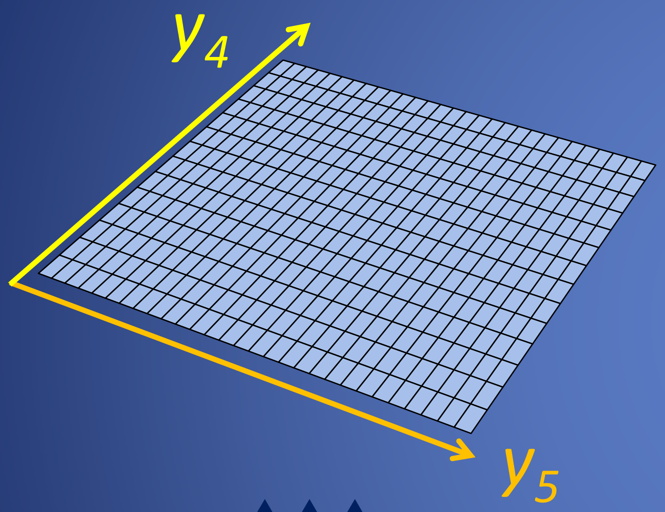
$$\lambda(x,y) = \sum_n \chi_n(x) \otimes \psi_n(y),$$

$$A_m(x,y) = \sum_n \varphi_{n,m}(x) \otimes \phi_{n,m}(y)$$

Magnetic flux on T^2

$$B = F_{45} = 2\pi M$$

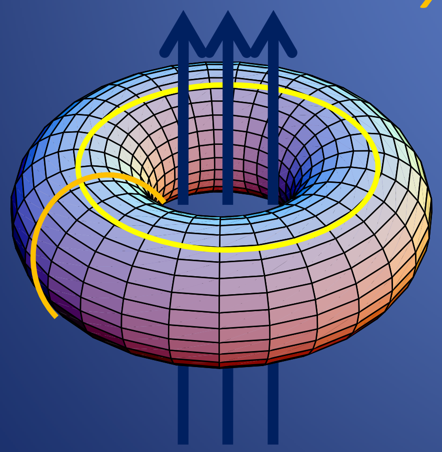
M = integer (Dirac quantization condition)



$$A_4 = 0\,, \quad A_5 = 2\pi M y_4$$

$$A_m(y_4+1, y_5) = A_m(y_4, y_5) + \partial_m \chi_4$$
$$A_m(y_4, y_5+1) = A_m(y_4, y_5) + \partial_m \chi_5$$

$$\psi(y_4+1, y_5) = e^{iq\chi_4}\psi(y_4, y_5)$$
$$\psi(y_4, y_5+1) = e^{iq\chi_5}\psi(y_4, y_5)$$



$\psi = \psi_0$: *zero-mode*

$$\chi_4 = 2\pi M y_5,$$
$$\chi_5 = 0.$$

Zero-mode eigenfunctions

D. Cremades, L.E. Ibanez & F. Marchesano, JHEP 0405 (2004) 079

From the 2nd b.c.

$$\psi_\pm(y_4, y_5) = \sum_n c_{\pm,n}(y_4) e^{2\pi i n y_5}$$

Then the Dirac eq. with *qM > 0* results in

$$c_{+,n}(y_4) = k_{+,n} e^{-\pi q M y_4^2 + 2\pi n y_4}$$

Finally the 1st b.c. determines

$$k_{+,n} = a_n e^{-\pi n^2/(qM)} \qquad a_{n+qM} = a_n$$

Zero-mode eigenfunctions

D. Cremades, L.E. Ibanez & F. Marchesano, JHEP 0405 (2004) 079

Relabeling $n = n' M + j$, $j = 0, 1, 2, \ldots, |M|-1$

$$\begin{cases} \psi_+^j = \Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi) \\ \psi_- = 0 : \quad \text{no normalizable zero-modes} \end{cases}$$

where $$\vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi) = \sum_n e^{-M\pi(n+j/M)^2 + 2\pi(n+j/M)M(y_4+iy_5)}$$

is the Jacobi theta function

Properties of zero-modes

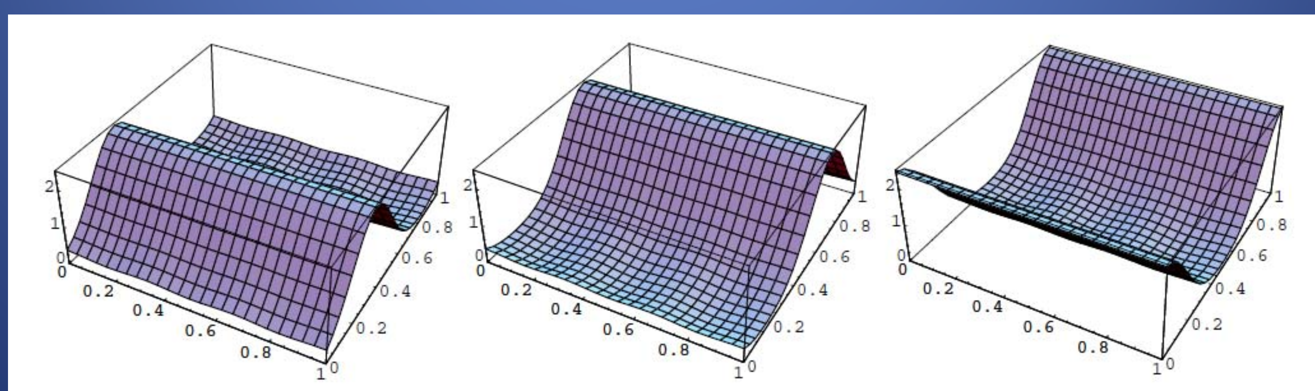
D. Cremades, L.E. Ibanez & F. Marchesano, JHEP 0405 (2004) 079

$|M|$ chiral zero-modes *$j = 0, 1, 2, \ldots, |M|-1$*

$$\psi_+^j = \Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi)$$

$\psi_- = 0$: no normalizable zero-modes

Wavefunction localization $|\psi_+^j|^2$, $|M|=3$

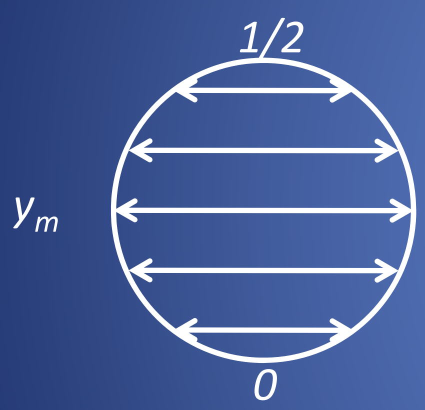


Zero-modes on T^2/Z_2

T. Kobayashi, H. Ohki & H. A., JHEP 0809 (2008) 043

Orbifold by Z_2 projection operator P $(P^2=1)$

$$\lambda_\pm(x,-y) = \pm P\lambda_\pm(x,y)P^{-1}$$



$P\lambda P^{-1} = +\lambda$

$P\lambda P^{-1} = -\lambda$

$$\psi^j_{even}(y) = \tfrac{1}{\sqrt{2}}\left(\psi^j(y) + \psi^{M-j}(y)\right)$$
$$\psi^j_{odd}(y) = \tfrac{1}{\sqrt{2}}\left(\psi^j(y) - \psi^{M-j}(y)\right)$$

of zero-modes

M	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4

10D SYM on magnetized tori

Basic features

- Degenerated chiral zero-modes appear
- The degeneracy is determined by the number of fluxes
- 4D Yukawa couplings are calculable

D. Cremades, L.E. Ibanez & F. Marchesano, JHEP 0405 (2004) 079

Applications

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10D SYM theory

The action

$$S = \int d^{10}X \sqrt{-G} \frac{1}{g^2} \mathrm{Tr}\left[-\frac{1}{4}F^{MN}F_{MN} + \frac{i}{2}\bar{\lambda}\Gamma^M D_M \lambda\right]$$

$$\begin{aligned} F_{MN} &= \partial_M A_N - \partial_N A_M - i[A_M, A_N], \\ D_M \lambda &= \partial_M \lambda - i[A_M, \lambda], \end{aligned}$$

10D vector ： A_M （$M = 0, 1, 2, \ldots, 9$）

10D Majorana-Weyl spinor ： λ $\lambda^C = \lambda$ $\Gamma\lambda = +\lambda$

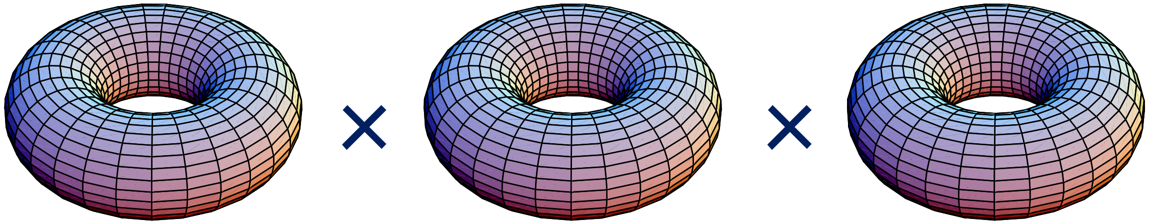
10D SYM theory on T^6

The torus compactification $T^2 \times T^2 \times T^2$

$$S = \int d^{10}X \sqrt{-G} \frac{1}{g^2} \mathrm{Tr} \left[-\frac{1}{4} F^{MN} F_{MN} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda \right]$$

$$X^M = (x^\mu, y^m) \qquad \mu = 0,1,2,3 \qquad m = 4, \ldots, 9$$

$$y^m \sim y^m + 2$$



$$z^i \equiv \frac{1}{2}(y^{2+2i} + \tau_i y^{3+2i}), \qquad z^{\bar{i}} \equiv (z^i)^*, \qquad i = 1,2,3$$

Field contents on T^6

10D vector : $A_M = (A_\mu, A_m) = (A_\mu, A_i)$ $i = 1,2,3$

$$A_i \equiv -\frac{1}{\mathrm{Im}\,\tau_i}(\tau_i^* A_{2+2i} - A_{3+2i}), \qquad \bar{A}_{\bar{i}} \equiv (A_i)^\dagger$$

10D Majorana-Weyl spinor : $\lambda = (\lambda_0, \lambda_i)$

Field contents on T^6

10D vector : $A_M = (A_\mu, A_m) = (A_\mu, A_i)$ $i = 1,2,3$

$$A_i \equiv -\frac{1}{\mathrm{Im}\,\tau_i}(\tau_i^* A_{2+2i} - A_{3+2i}), \qquad \bar{A}_{\bar{i}} \equiv (A_i)^\dagger$$

10D Majorana-Weyl spinor : $\lambda = (\lambda_0, \lambda_i)$

$\mathcal{N} = 1$ supermultiplets (superfields):

$$V = \{A_\mu, \lambda_0\}, \qquad \phi_i = \{A_i, \lambda_i\}$$

U(N) adjoints

Magnetic flux background

Abelian flux & Wilson-line in $U(N)$ adjoint matrix

$$\langle A_i \rangle = \frac{\pi}{\mathrm{Im}\,\tau_i} \left(M^{(i)}\, \bar{z}_i + \bar{\zeta}_i \right)$$

$$M^{(i)} = \mathrm{diag}(M_1^{(i)}, M_2^{(i)}, \ldots, M_N^{(i)}),$$ Magnetic fluxes

$$\zeta_i = \mathrm{diag}(\zeta_1^{(i)}, \zeta_2^{(i)}, \ldots, \zeta_N^{(i)}),$$ Wilson-lines

$$M_a^{(i)} \neq M_b^{(i)} \quad {}^{\forall} a, b \quad \Rightarrow \quad U(N) \to U(1)^N$$

Abelian magnetic flux in $U(N)$ SYM

- $U(N) \to U(N_a) \times U(N_b) \times U(N_c)$

$$F_{45} = 2\pi \begin{pmatrix} M_a^{(1)} \mathbf{1}_{N_a \times N_a} & & 0 \\ & M_b^{(1)} \mathbf{1}_{N_b \times N_b} & \\ 0 & & M_c^{(1)} \mathbf{1}_{N_c \times N_c} \end{pmatrix}$$

$M_1 + M_2 = M_3$

$M_1 = M_a - M_b$, $M_2 = M_c - M_a$, $M_3 = M_c - M_b$

- Zero-modes in adjoint field

$$\lambda_+(x,y) = \begin{pmatrix} \lambda_a(x) & \psi_+^i(y) L_{ab}^i(x) & 0 \\ 0 & \lambda_b(x) & 0 \\ \psi_+^j(y) R_{ca}^j(x) & \psi_+^k(y) H_{cb}^k(x) & \lambda_c(x) \end{pmatrix}$$

for $M_3 > 0$

$i = 0, 1, \ldots, |M_1|-1$
$j = 0, 1, \ldots, |M_2|-1$
$k = 0, 1, \ldots, |M_3|-1$

PHENOMENOLOGICAL ASPECTS

MSSM-LIKE MODELS (VISIBLE SECTOR)

10D $U(8)$ SYM model on T^6

T. Kobayashi, H. Ohki, K. Sumita & H. A., NPB 863 (2012) 1-18

Magnetic fluxes $U(8) \rightarrow U(4)_C \times U(2)_L \times U(2)_R$

$$F_{2+2r,3+2r} = 2\pi \begin{pmatrix} M_C^{(r)} \mathbf{1}_4 & & \\ & M_L^{(r)} \mathbf{1}_2 & \\ & & M_R^{(r)} \mathbf{1}_2 \end{pmatrix}$$

$r = 1,2,3$

10D $U(8)$ SYM model on T^6

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$r = 1,2,3$

Wilson-lines $\rightarrow U(3)_C \times U(2)_L \times U(1)_{C'} \times U(1)_{R'} \times U(1)_{R''}$

$$\zeta_r = \begin{pmatrix} \zeta_C^{(r)} \mathbf{1}_3 & & & & \\ & \zeta_{C'}^{(r)} & & & \\ & & \zeta_L^{(r)} \mathbf{1}_2 & & \\ & & & \zeta_{R'}^{(r)} & \\ & & & & \zeta_{R''}^{(r)} \end{pmatrix}$$

10D $U(8)$ SYM model on T^6

T. Kobayashi, H. Ohki, K. Sumita & H. A., NPB 863 (2012) 1-18

Flux ansatz

$$
\begin{aligned}
(M_C^{(1)}, M_L^{(1)}, M_R^{(1)}) &= (0, +3, -3), \\
(M_C^{(2)}, M_L^{(2)}, M_R^{(2)}) &= (0, -1, 0), \\
(M_C^{(3)}, M_L^{(3)}, M_R^{(3)}) &= (0, 0, +1),
\end{aligned}
$$

→ Three generations of quarks and leptons and six generations of Higgs

SUSY conditions

$$
\begin{aligned}
h^{\bar{i}j}\left(\bar{\partial}_{\bar{i}}\langle A_j\rangle + \partial_j\langle \bar{A}_{\bar{i}}\rangle\right) &= 0, \\
\epsilon^{\mathrm{jkl}} e_{\mathrm{k}}{}^{k} e_{\mathrm{l}}{}^{l} \partial_k \langle A_l\rangle &= 0,
\end{aligned}
$$

⇔ $\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = \mathcal{A}^{(1)}/\mathcal{A}^{(3)} = 3$

Matter zero-modes on T^6

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

Zero-modes in ϕ_i

$$\phi_1^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} \Omega_C^{(1)} & \Xi_{CC'}^{(1)} & 0 & \Xi_{CR'}^{(1)} & \Xi_{CR''}^{(1)} \\ \Xi_{C'C}^{(1)} & \Omega_{C'}^{(1)} & 0 & \Xi_{C'R'}^{(1)} & \Xi_{C'R''}^{(1)} \\ \hline \Xi_{LC}^{(1)} & \Xi_{LC'}^{(1)} & \Omega_L^{(1)} & H_u^K & H_d^K \\ \hline 0 & 0 & 0 & \Omega_{R'}^{(1)} & \Xi_{R'R''}^{(1)} \\ 0 & 0 & 0 & \Xi_{R''R'}^{(1)} & \Omega_{R''}^{(1)} \end{array}\right) \qquad \phi_2^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} \Omega_C^{(2)} & \Xi_{CC'}^{(2)} & Q^I & 0 & 0 \\ \Xi_{C'C}^{(2)} & \Omega_{C'}^{(2)} & L^I & 0 & 0 \\ \hline 0 & 0 & \Omega_L^{(2)} & 0 & 0 \\ \hline 0 & 0 & 0 & \Omega_{R'}^{(2)} & \Xi_{R'R''}^{(2)} \\ 0 & 0 & 0 & \Xi_{R''R'}^{(2)} & \Omega_{R''}^{(2)} \end{array}\right)$$

$$\phi_3^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} \Omega_C^{(3)} & \Xi_{CC'}^{(3)} & 0 & 0 & 0 \\ \Xi_{C'C}^{(3)} & \Omega_{C'}^{(3)} & 0 & 0 & 0 \\ \hline 0 & 0 & \Omega_L^{(3)} & 0 & 0 \\ \hline U^J & N^J & 0 & \Omega_{R'}^{(3)} & \Xi_{R'R''}^{(3)} \\ D^J & E^J & 0 & \Xi_{R''R'}^{(3)} & \Omega_{R''}^{(3)} \end{array}\right)$$

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Matter zero-modes on T^6

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

Zero-modes in ϕ_i

Six generations of Higgs (K = 1,2,…,6)

Three generations of left-handed quarks and leptons (I = 1,2,3)

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Three generations of right-handed quarks and leptons (J = 1,2,3)

Matter zero-modes on T^6/Z_2

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

Zero-modes in ϕ_i

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Three generations of right-handed quarks and leptons (J = 1,2,3)

on orbifold T^6/Z_2

Matter zero-modes on T^6/Z_2

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

T^6/Z_2 orbifold

$$
\begin{aligned}
V(x, y_m, -y_n) &= +PV(x, y_m, +y_n)P^{-1}, \\
\phi_1(x, y_m, -y_n) &= +P\phi_1(x, y_m, +y_n)P^{-1}, \\
\phi_2(x, y_m, -y_n) &= -P\phi_2(x, y_m, +y_n)P^{-1}, \\
\phi_3(x, y_m, -y_n) &= -P\phi_3(x, y_m, +y_n)P^{-1},
\end{aligned}
$$

$^\forall m = 4, 5$ and $^\forall n = 6, 7, 8, 9$

does not break SUSY preserved by the flux

$$
P_{ab} = \begin{pmatrix} -\mathbf{1}_4 & 0 & 0 \\ 0 & +\mathbf{1}_2 & 0 \\ 0 & 0 & +\mathbf{1}_2 \end{pmatrix}
$$

projects out many exotic modes without affecting MSSM contents

Matter zero-modes on T^6/Z_2

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

Zero-modes in ϕ_i on orbifold T^6/Z_2

$$\phi_1^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} \Omega_C^{(1)} & \Xi_{CC'}^{(1)} & 0 & 0 & 0 \\ \Xi_{C'C}^{(1)} & \Omega_{C'}^{(1)} & 0 & 0 & 0 \\ \hline 0 & 0 & \Omega_L^{(1)} & H_u^K & H_d^K \\ \hline 0 & 0 & 0 & \Omega_{R'}^{(1)} & \Xi_{R'R''}^{(1)} \\ 0 & 0 & 0 & \Xi_{R''R'}^{(1)} & \Omega_{R''}^{(1)} \end{array}\right) \qquad \phi_2^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} 0 & 0 & Q^I & 0 & 0 \\ 0 & 0 & L^I & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\phi_3^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline U^J & N^J & 0 & 0 & 0 \\ D^J & E^J & 0 & 0 & 0 \end{array}\right)$$

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$$\phi_3^{\mathcal{I}_{ab}} = \left(\begin{array}{cc|c|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline U^J & N^J & 0 & 0 & 0 \\ D^J & E^J & 0 & 0 & 0 \end{array}\right)$$

Flavor structure

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

	Sample values	Observed (GeV)
(m_u, m_c, m_t)	$(3.1 \times 10^{-3}, 1.01, 1.70 \times 10^{2})$	$(2.3 \times 10^{-3}, 1.28, 1.74 \times 10^{2})$
(m_d, m_s, m_b)	$(2.8 \times 10^{-3}, 1.48 \times 10^{-1}, 6.46)$	$(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18)$
(m_e, m_μ, m_τ)	$(4.68 \times 10^{-4}, 5.76 \times 10^{-2}, 3.31)$	$(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78)$
$\lvert V_{\mathrm{CKM}} \rvert$	$\begin{pmatrix} 0.98 & 0.21 & 0.0023 \\ 0.21 & 0.98 & 0.041 \\ 0.011 & 0.040 & 1.0 \end{pmatrix}$	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$

Particle Data Group Collaboration (Beringer, J. et al.)
Phys.Rev. D86 (2012) 010001

A semi-realistic pattern from non-hierarchical paramters

Flavor structure

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

	Sample values	Observed
$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$	$(3.6 \times 10^{-19}, 8.8 \times 10^{-12}, 2.7 \times 10^{-11})$	$< \ 2 \times 10^{-9}$ (GeV)
$\lvert m_{\nu_1}^2 - m_{\nu_2}^2 \rvert$	7.67×10^{-23}	7.50×10^{-23}
$\lvert m_{\nu_1}^2 - m_{\nu_3}^2 \rvert$	7.12×10^{-22}	2.32×10^{-21}
$\lvert V_{\mathrm{PMNS}} \rvert$	$\begin{pmatrix} 0.85 & 0.46 & 0.25 \\ 0.50 & 0.59 & 0.63 \\ 0.15 & 0.66 & 0.73 \end{pmatrix}$	$\begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.51 & 0.58 & 0.64 \\ 0.26 & 0.61 & 0.75 \end{pmatrix}$

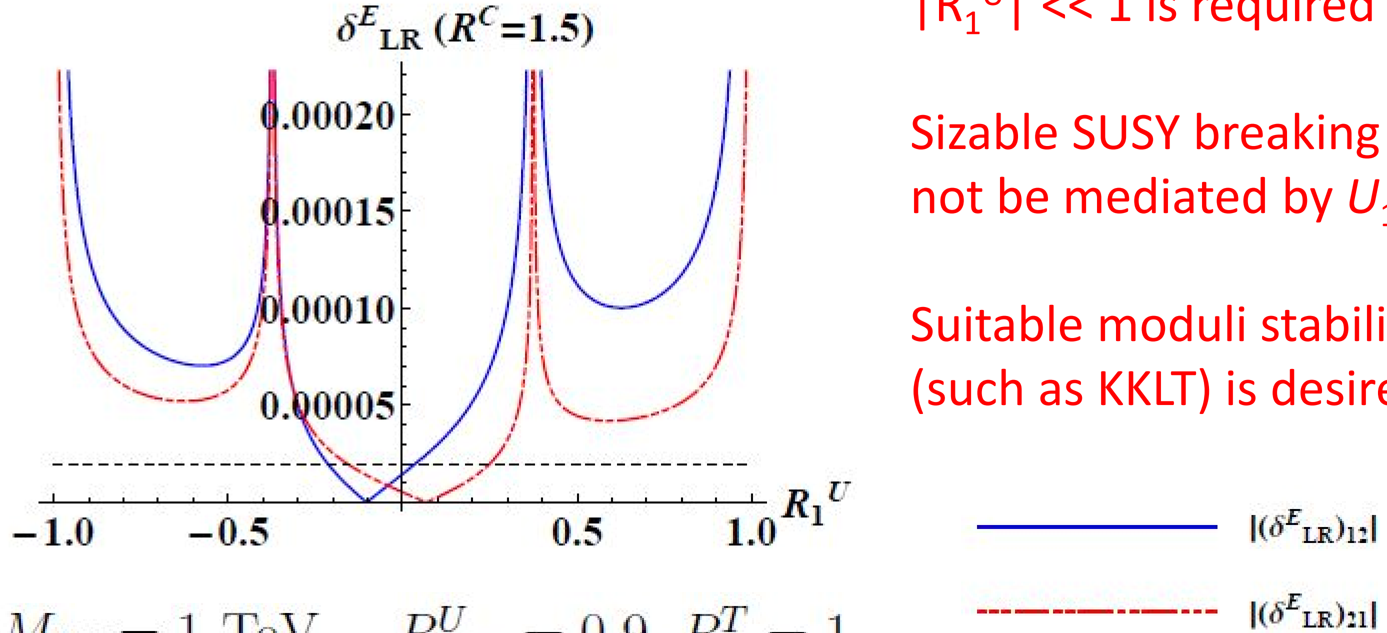
Particle Data Group Collaboration (Beringer, J. et al.)
Phys.Rev. D86 (2012) 010001

A semi-realistic pattern from non-hierarchical paramters

Implications on SUSY particles

T. Kobayashi, H. Ohki, A. Oikawa, K. Sumita & H. A., NPB 870 (2013) 30-54

The most stringent bound from $\mu \to e\gamma$ on δ^E_{LR}



$M_{SB} = 1$ TeV $\quad R^U_{r\neq 1} = 0.9,\ R^T_r = 1$

$|R_1{}^U| << 1$ is required

Sizable SUSY breaking can not be mediated by U_1

Suitable moduli stabilization (such as KKLT) is desired

Some prospects

- Flavor symmetry
  - Delta(27) with SUSY fluxes

    ``Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models,''
    K.-S. Choi, T. Kobayashi, H. Ohki & H. A., NPB 820 (2009) 317-333

  - Gaussian Froggatt-Nielsen with non-SUSY fluxes

    ``Gaussian Froggatt-Nielsen mechanism on magnetized orbifolds,''
    T. Kobayashi, K. Sumita, Y. Tatsuta & H. A., PRD 90 (2014) 105006

- SUSY model with non-SUSY fluxes

  ``Supersymmetric models on magnetized orbifolds with flux-induced Fayet-Iliopoulos terms,''
  T. Kobayashi, K. Sumita & H. A., arXiv:1610.07730

$$
\begin{aligned}
h^{\bar{i}j}\left(\bar{\partial}_{\bar{i}}\langle A_j\rangle + \partial_j\langle \bar{A}_{\bar{i}}\rangle\right) &\neq 0, \\
\epsilon^{\mathrm{jkl}}{e_{\mathrm{k}}}^{k}{e_{\mathrm{l}}}^{l}\partial_k\langle A_l\rangle &= 0,
\end{aligned}
$$

Some prospects

- *U*(8) can be embedded into *SO*(32)

  ⇔ Type I (IIB with D9) or Heterotic construction?

  ``Realistic three-generation models from SO(32) heterotic string theory,''
  T. Kobayashi, H. Otsuka, Y. Takano and H. A., JHEP 1509 (2015) 056

- 8D (6D) SYM is enough for the flavor structure

  ``Superfield description of (4+2n)-dimensional SYM theories and their mixtures on magnetized tori,''
  T. Horie, K. Sumita & H. A., NPB 900 (2015) 331-365

$$
\begin{aligned}
(M_C^{(1)}, M_L^{(1)}, M_R^{(1)}) &= (0, +3, -3),\\
(M_C^{(2)}, M_L^{(2)}, M_R^{(2)}) &= (0, -1, 0),\\
(M_C^{(3)}, M_L^{(3)}, M_R^{(3)}) &= (0, 0, +1),
\end{aligned}
$$

→ Three generations of quarks and leptons and six generations of Higgs

⇔ Type IIB D3/D7 or D5/D9 construction?

PHENOMENOLOGICAL ASPECTS

DSB MODELS (HIDDEN SECTOR)

Hidden sector models

- Magnetized SYM in higher-dim.

  → 4D chiral gauge theories with flavors

  will be applied to

  not only the visible (SM) sector

  but also the hidden (DSB or moduli stab.) sectors

10D $U(N)$ SYM model on T^6

T. Kobayashi, K. Sumita & H. A., NPB 911 (2016) 606-622

Fluxes leading to $U(N) \to U(N_C) \times U(N_X) \times U(N_Y)$

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Zero-modes in ϕ_i

$$\phi_1 = \begin{pmatrix} \Xi_1 & 0 & 0 \\ \tilde{Q}' & \Xi_1' & 0 \\ Q & 0 & \Xi_1'' \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \Xi_2 & \tilde{Q} & 0 \\ 0 & \Xi_2' & 0 \\ 0 & S' & \Xi_2'' \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} \Xi_3 & 0 & Q' \\ 0 & \Xi_3' & S \\ 0 & 0 & \Xi_3'' \end{pmatrix}$$

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Zero-modes in ϕ_i

$$\phi_1 = \begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ Q & 0 & \times \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \times & \tilde{Q} & 0 \\ 0 & \times & 0 \\ 0 & \times & \times \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & S \\ 0 & 0 & \times \end{pmatrix}$$

$$P_{+--} = \begin{pmatrix} + & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & - \end{pmatrix} \quad P_{+-+} = \begin{pmatrix} + & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & + \end{pmatrix}$$

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Superpotential

$$W = SQ\tilde{Q}$$

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Zero-modes in ϕ_i $\to$ $U(N_C)$ SYM with N_F flavors $Q, \tilde{Q}$

$$\phi_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Q & 0 & 0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 & \tilde{Q} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & S \\ 0 & 0 & 0 \end{pmatrix}$$

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Superpotential for $N_C > N_F$

$$W = SQ\tilde{Q} + C_{N_C,N_F}\left(\frac{\Lambda^{3N_C-N_F}}{\det Q\tilde{Q}}\right)^{1/(N_C-N_F)}$$

→ Dynamical SUSY breaking (DSB)

I. Affleck, M. Dine and N. Seiberg, NPB 241 (1984) 493-534, PLB 137 (1984) 187

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T. Kobayashi, K. Sumita & H. A., NPB 911 (2016) 606-622

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Zero-modes in ϕ_i → $U(N_C)$ SYM with N_F flavors $Q, \tilde{Q}$

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→ Dynamical SUSY breaking (DSB)

I. Affleck, M. Dine and N. Seiberg, NPB 241 (1984) 493-534, PLB 137 (1984) 187

Some prospects

- DSB seems to occur
- $U(8+N) \rightarrow U(8)_{\text{visible}} \times U(N)_{\text{hidden}}$
  - Viable flux/orbifold configurations?
  - Messengers → Gauge mediation
- Moduli stabilization sector?

SUMMARY

SYM on magnetized tori

- Magnetic fluxes determine almost everything :

  Gauge syms, chirality, # of gens, hierarchies,

  moduli-mediated sparticle spectra, DSB, …

  Phenomenological aspects are quite interesting
- Not God but (static) dynamics could yield the hierarchy of elements
- Low-energy effective theory of superstrings