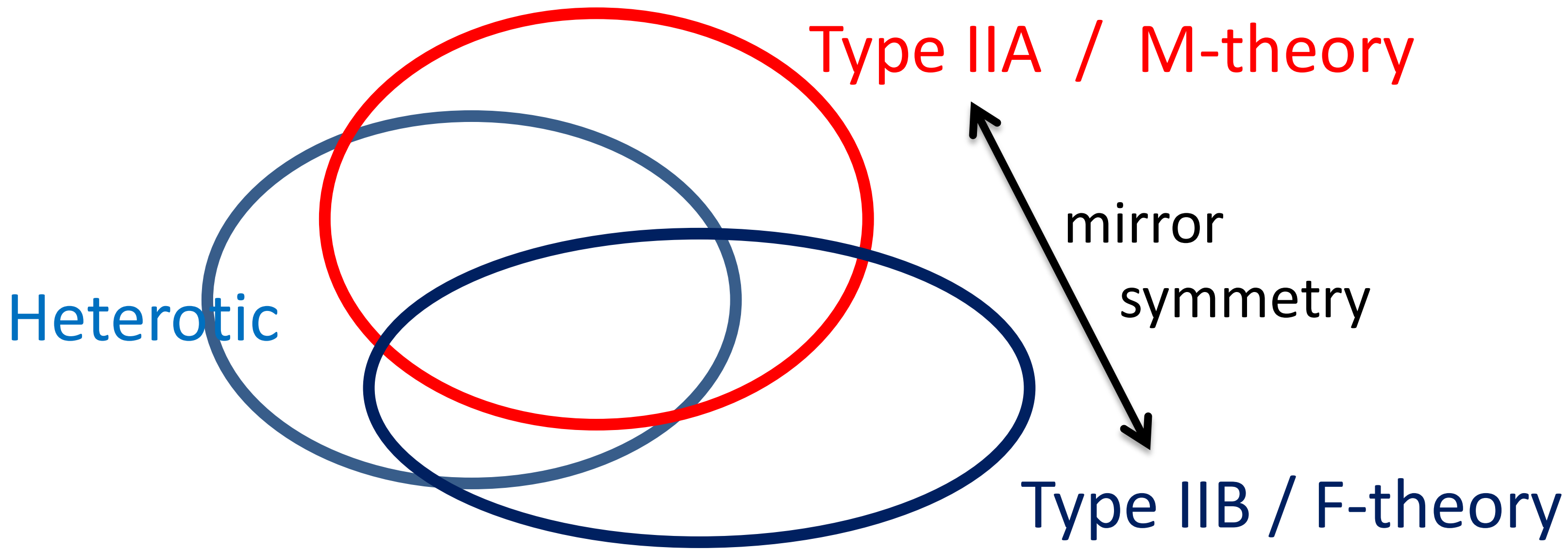
Heterotic—IIA duality map of discrete data

Dec. 01, ’16 String theory in greater Tokyo

Taizan Watari at IPMU

arXiv:1604.06437 with A.P.Braun (Oxford)

- Heterotic—Type II duality



  - Profound phys / math from mirror symmetry
  - How about Heterotic---Type II duality?

Yau-Zaslow conjecture '95, Katz--Klemm—Vafa conjecture,
Harvey-Moore '95, Maulik—Pandharipande '07, ... and maybe more.

- Duality Het $\Longleftrightarrow$ IIA @6D

T^4 Narain — K3 (B, metric)

$$\mathrm{Isom}(\mathrm{II}_{4,20})\backslash O(4,20;\mathbb{R})/O(4)\times O(20)$$

  - 16 SUSY charges preserved
  - lift to 7D Het/ T^3 $\Longleftrightarrow$ M-theory/ K3

Seiberg '88, Aspinwall Morrison '94, Vafa Witten '94, Harvey—Strominger '95, Bershadsky-Sadov-Vafa '95 …

- Duality Het $\Longleftrightarrow$ F-theory @8D

T^2 Narain — elliptic K3 cpx. str.

$$\mathrm{Isom}(\mathrm{II}_{2,18})\backslash O(2,18;\mathbb{R})/O(2)\times O(18)$$

- Duality Het $\Longleftrightarrow$ IIA @6D

T^4 Narain K3

$$\mathrm{Isom}(\mathrm{II}_{4,20})\backslash O(4,20;\mathbb{R}) / O(4)\times O(20)$$

- 6D eff. theories w/ (1,1) SUSY

fibred adiabatically over $\mathbb{P}^1$ $\longrightarrow$ 4D N=2 SUSY.

$$\mathrm{Het} / "T^2\times" \mathrm{K3} \Longleftrightarrow \mathrm{IIA} / \text{K3-fib.} CY_3 = M$$

Kachru Vafa ’95 Klemm Lerche Mayr ’95
Ferrara et.al. ’95, Vafa Witten ’95, …….

- fibre adiabatically over $\mathbb{P}^1$
  - first step: specify a lattice polarization of K3 (IIA).

	$[U \oplus \Lambda_S] \otimes \mathbb{C}$	$\oplus \quad \Lambda_T \otimes \mathbb{C}$	$\subset \mathrm{II}_{4,20} \otimes \mathbb{C}$
$(k^8 + ik^9)$	"fixed" over $\mathbb{P}^1$		$(B + iJ)[K3]$
$(k^6 + ik^7)$		vary over $\mathbb{P}^1$	$\Omega(\mathrm{K3})$

  - second: two aspects to study
    - further discrete choices in fibration. Part I
    - degeneration of fibre. not adiabatic. Part II

Part I:
Duality Dictionary of Discrete Data

- fibre adiabatically over $\mathbb{P}^1$
  - first step: specify a lattice polarization of K3 (IIA).

	$[U \oplus \Lambda_S] \otimes \mathbb{C}$	$\oplus$ $\Lambda_T \otimes \mathbb{C}$	$\subset \mathrm{II}_{4,20} \otimes \mathbb{C}$
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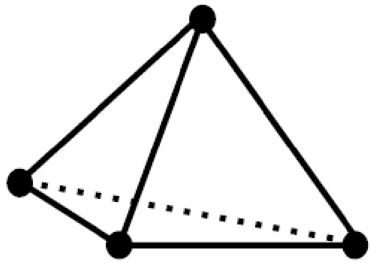
$(k^6 + ik^7)$		vary over $\mathbb{P}^1$	$\Omega(\mathrm{K3})$

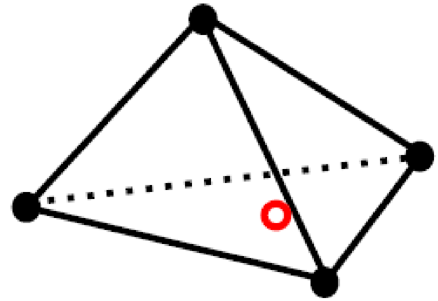
  - examples:
    - quartic K3 $(4) \subset \mathbb{CP}^3$ $\quad \Lambda_S = \langle +4 \rangle,$ $\quad \Lambda_T = U^{\oplus 2} \oplus E_8^{\oplus 2} \oplus \langle -4 \rangle.$
    - elliptic K3 $\quad \Lambda_S = U,$ $\quad \Lambda_T = U^{\oplus 2} \oplus E_8^{\oplus 2}.$

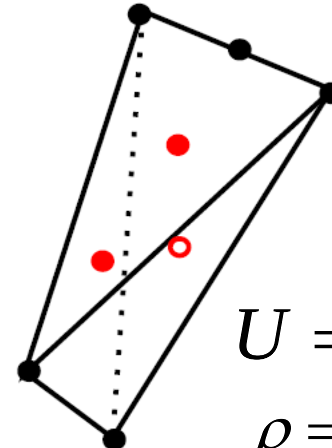
- Multiple choices of lattice-pol. K3 fibration

toric data (polytopes)

$\tilde{\Delta}_{K3} =$

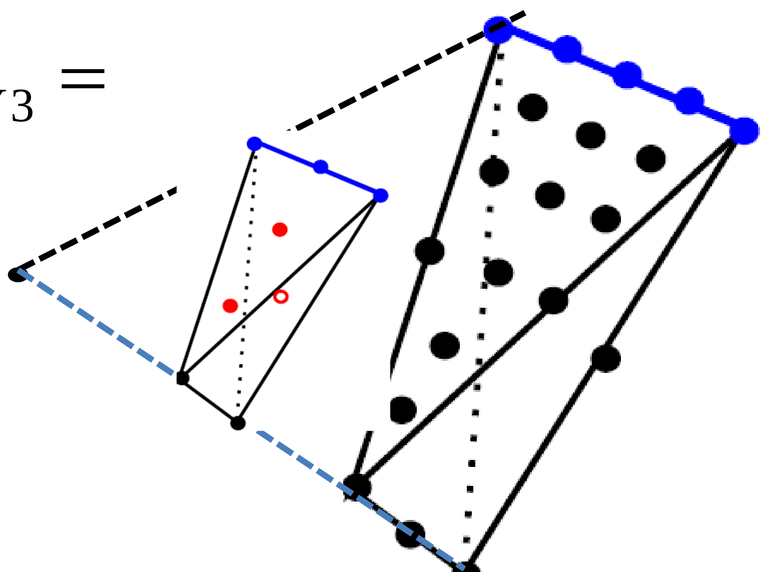






$\Lambda_S =$ $\langle +4 \rangle$ $\rho = 1$ $\langle +2 \rangle$ $\rho = 1$ $U = II_{1,1}$ $\rho = 2.$

$\tilde{\Delta}_{CY3} =$



Choose any one from

$$2\tilde{\Delta}_{K3} \cap \mathbb{Z}^{\oplus 3}$$

For $h^{1,1}(M) = \rho + 1,$
blue points only.

Candelas Font '96

- Multiple choices of lattice-pol. K3 fibration

toric data (polytopes)

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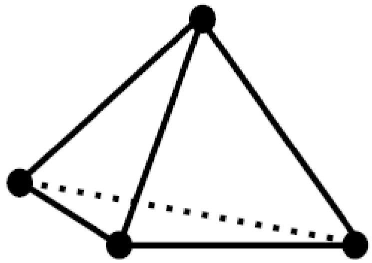

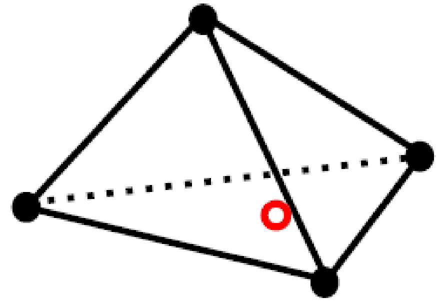

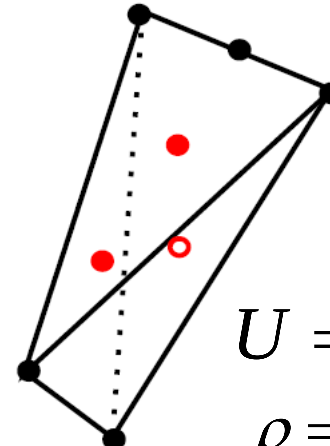

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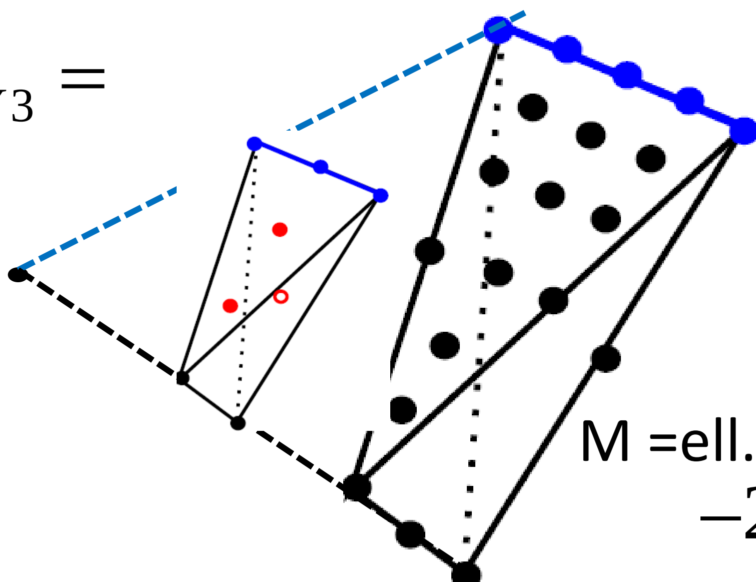

M =ell.fibr. over F_n
$-2 \leq n \leq +2.$

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Candelas Font ‘96

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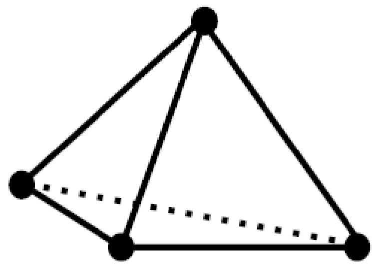

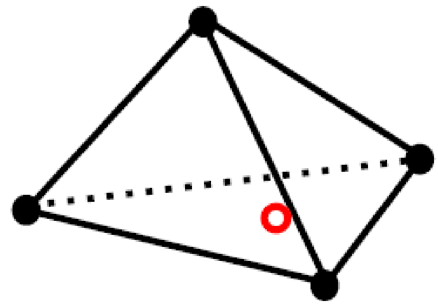

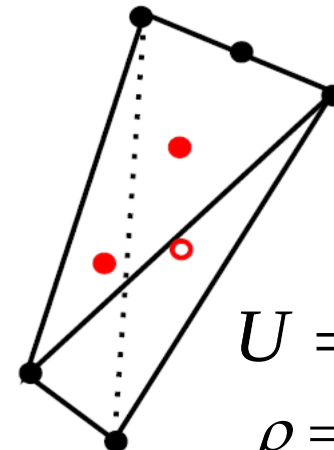

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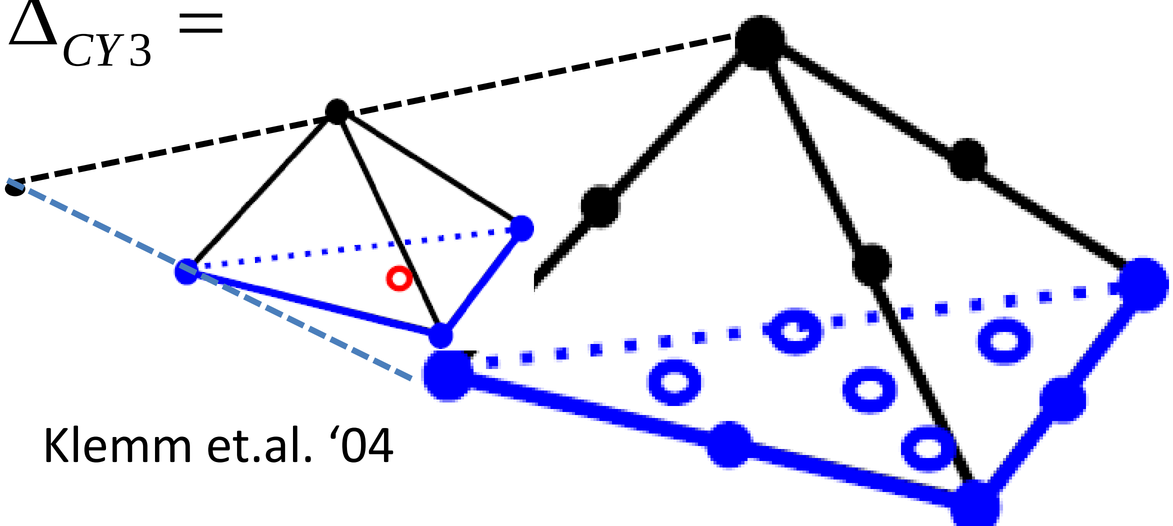

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blue points only.

Klemm et.al. '04

Candelas Font '96

- Multiple choices of lattice-pol. K3 fibration
  - 4319 choices (Λ_S, Λ_T) as toric hypersurface

Kreuzer Skarke '98

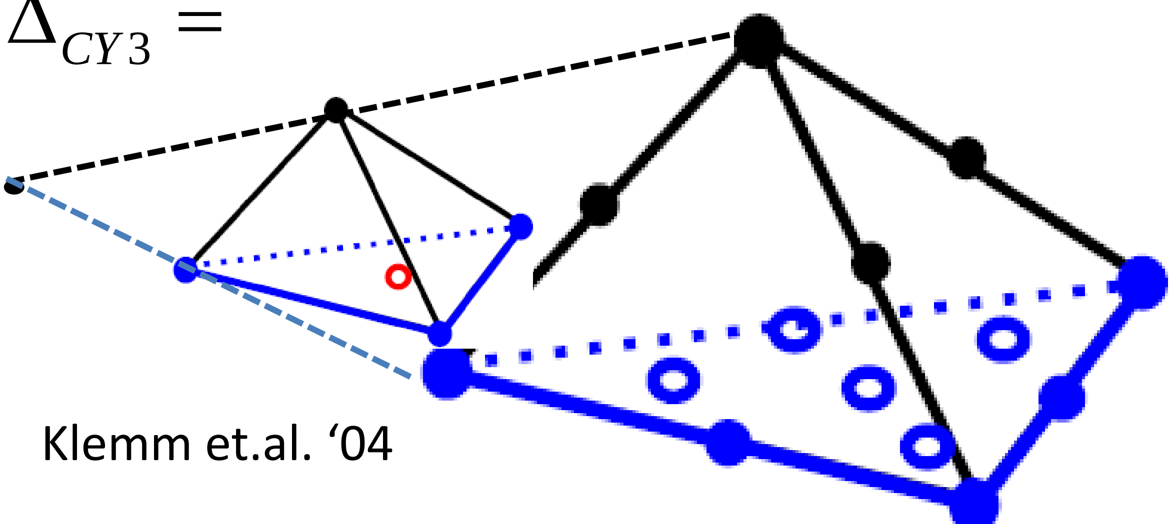
  - 3117 of them admit Λ_S-K3 fibration

with $h^{1,1}(M) = \rho + 1,$

  - 1983 of them come with multiple choices,
    - sometimes the same $h^{2,1}$, sometimes not.

...... in Type IIA language

$\tilde{\Delta}_{CY3} =$



Klemm et.al. '04

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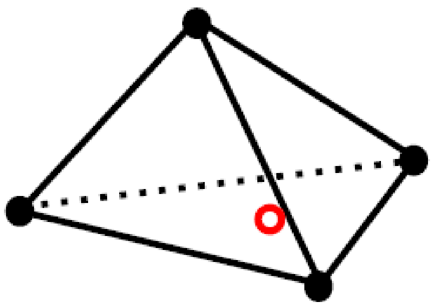
Candelas Font '96

- In the case of deg.2 K3 in the fibre,

$$\tilde{\Delta}_{K3} =$$



$$h^{1,1} + 1 = \#(vect) = 3$$
$$h^{2,1} + 1 = \#(hypr) = 129.$$

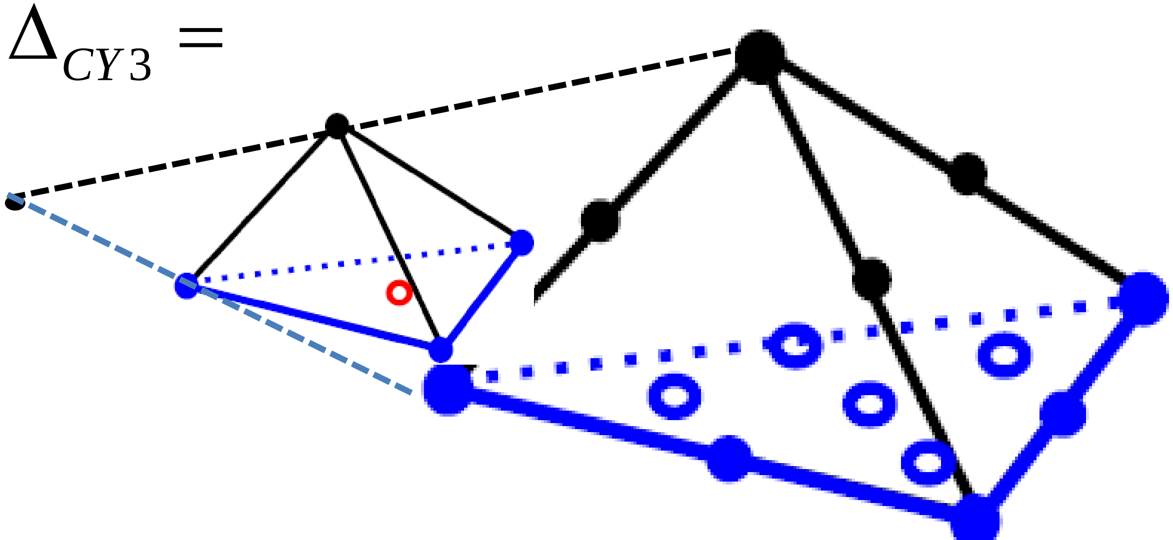
Kachru Vafa '95

$$\Lambda_S = \langle +2 \rangle$$
$$\rho = 1$$

Type IIA on CY3 $\Longleftrightarrow$ Het on "T2 x" K3 instanton 4+10+10

GW-inv of vert. classes + **<div.div.div.> intersection** $\Longleftrightarrow$ Het 1-loop threshold

$$\tilde{\Delta}_{CY3} =$$

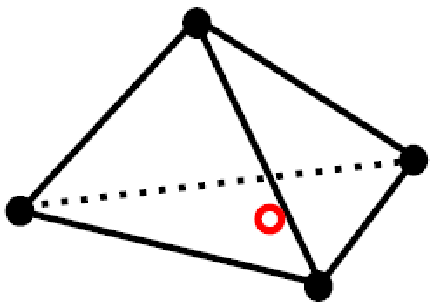


Kaplunovsky et.al., Antoniadis et.al. '95 Klemm et.al. '04

which one is dual?

- In the case of deg.2 K3 in the fibre,

$\tilde{\Delta}_{K3} =$



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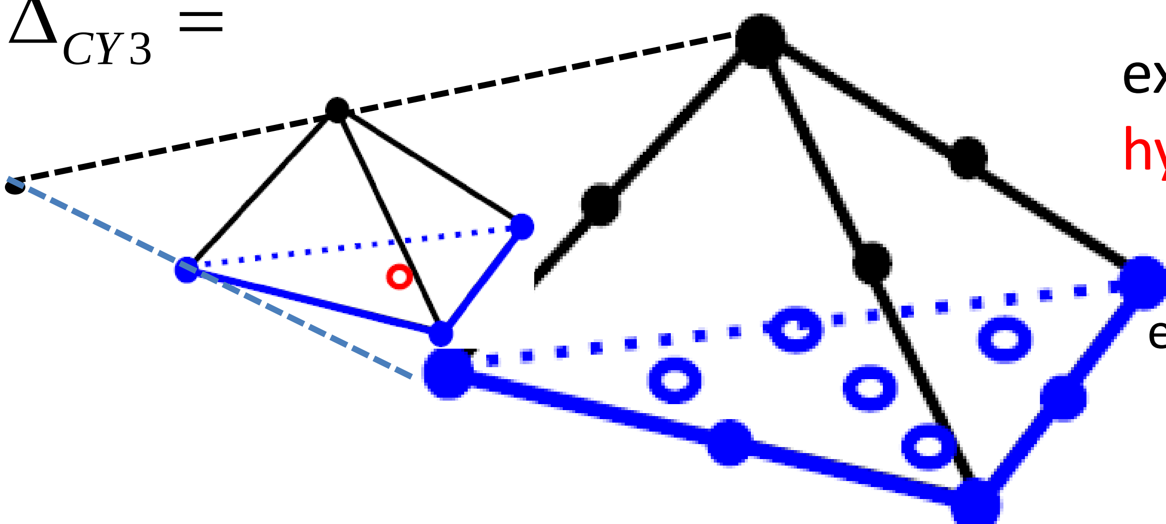


exploit detailed info. of hyper –mult. moduli space

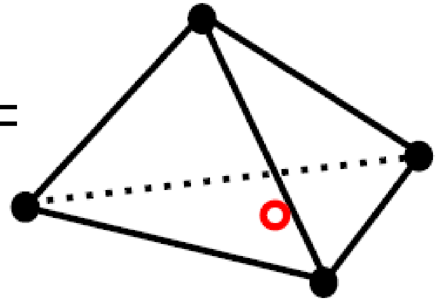
$$y^2 = F^{(6)}(X_2, X_3, X_4).$$

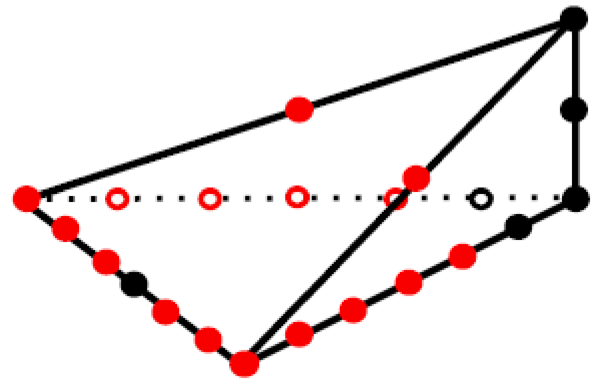
each coeff. → polynomial on $\mathbb{P}^1$

right DOFs for $E_8 \times E_8$ or not?

which one is dual?

$\tilde{\Delta}_{K3} =$





$= \Delta_{K3}$

$$
\begin{aligned}
y^2 = \quad & (a'_1 x_4^5 x_3 + a'_2 x_4^4 x_3^2 + a'_3 x_4^3 x_3^3 + a'_4 x_4^2 x_3^4 + a'_5 x_4 x_3^5 + a'_6 x_3^6) \\
& + (b'_1 x_4^3 x_3^2 + b'_2 x_4^2 x_3^3 + b'_3 x_4 x_3^4 + b'_4 x_3^5) + (c'_1 x_4 x_3^3 + c'_2 x_3^4) \\
& + x_4^6 + b_0 x_4^4 x_3 + c_0 x_4^2 x_3^2 + d_0 x_3^3 \\
& + (b_1 x_4^3 + b_2 x_4^2 + b_3 x_4 + b_4) x_3 + (c_1 x_4 + c_2) x_3^2 \\
& + (a_1 x_4^5 + a_2 x_4^4 + a_3 x_4^3 + a_4 x_4^2 + a_5 x_4 + a_6),
\end{aligned}
$$

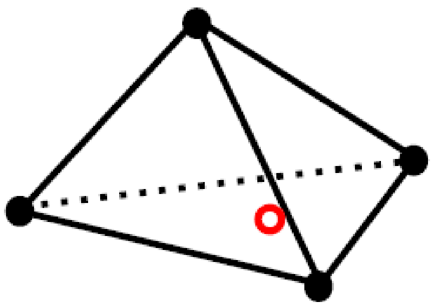
$$
\begin{aligned}
a_{r=1,\cdots,6} &\in \Gamma(\mathbb{P}^1_A; \mathcal{O}(12 + r(I_v - 12))), \\
b_{r=1,\cdots,4} &\in \Gamma(\mathbb{P}^1_A; \mathcal{O}(8 + r(I_v - 12))), \\
c_{r=1,2} &\in \Gamma(\mathbb{P}^1_A; \mathcal{O}(4 + r(I_v - 12))),
\end{aligned}
$$

Braun TW '16

- In the case of deg.2-K3 fibration

Braun TW '16

$\tilde{\Delta}_{K3} =$



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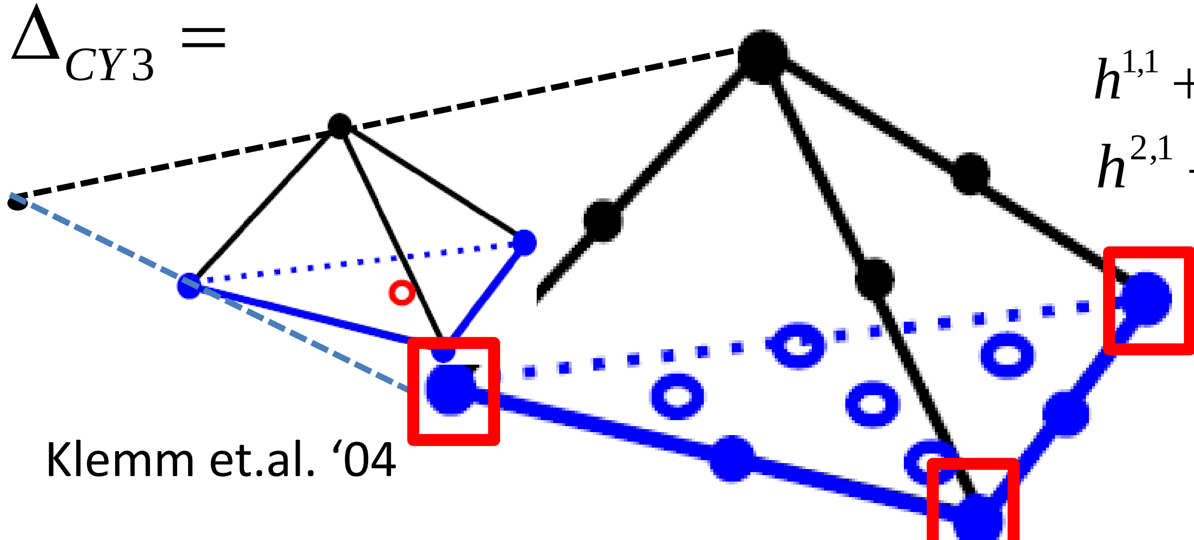

$$h^{1,1} + 1 = \#(vect) = 3$$

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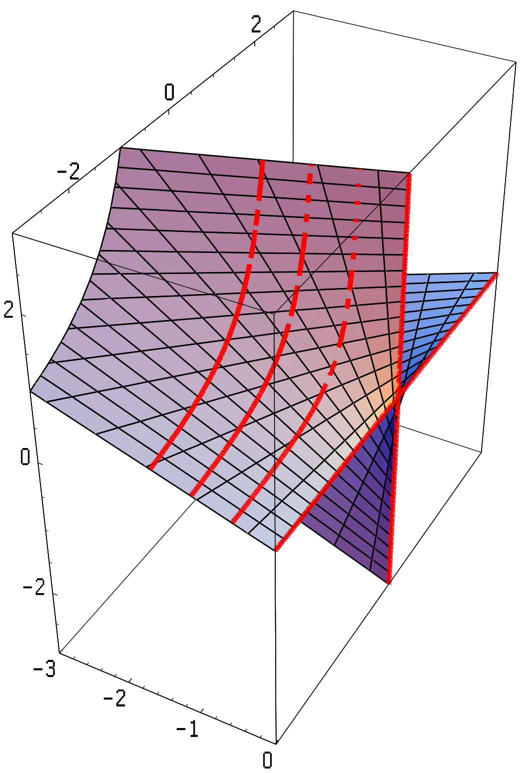
Others do not allow free 4+10+10 instanton interpret'n.

Klemm et.al. '04

Part II: degenerations of K3 and solitons

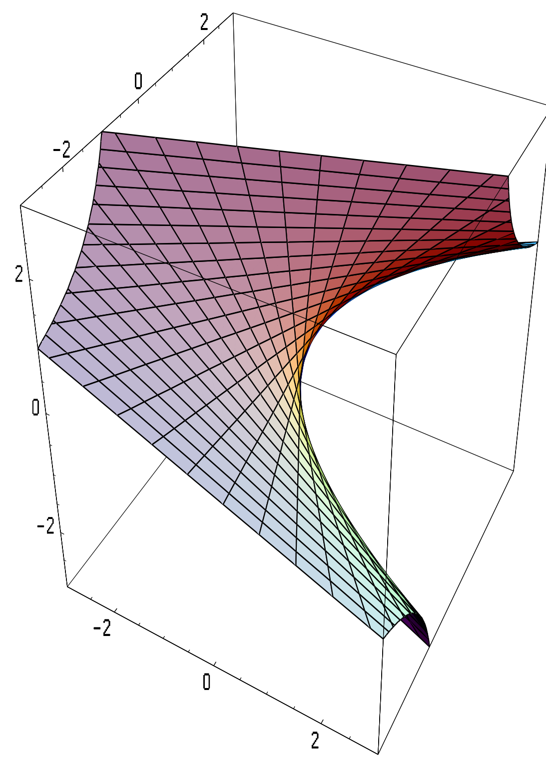


An example of degeneration

$$X = \{(x, y, t) \mid xy = t\}$$

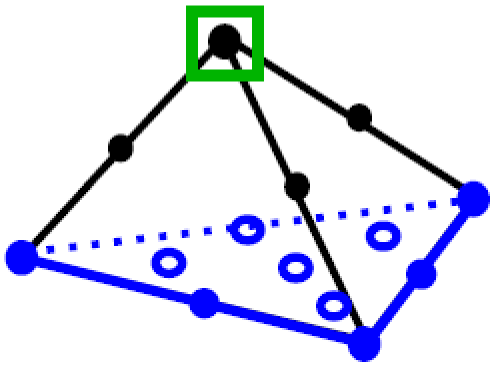
$$\downarrow$$

$$t \in \mathbb{C}$$



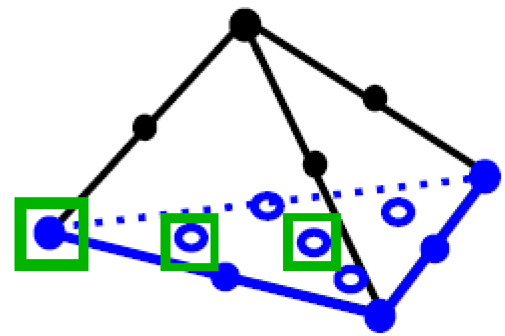
Friedman '84, …., Davis et.al. '13, Braun TW '16

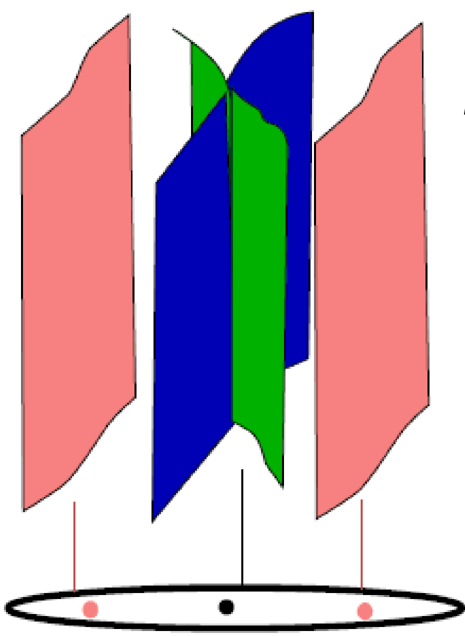
- Type IIA / M = deg-2 K3 fibr. over $\mathbb{P}^1$



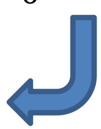
Add point(s) from

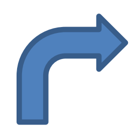
$$2\tilde{\Delta}_{K3} \cap \mathbb{Z}^{\oplus 3}$$



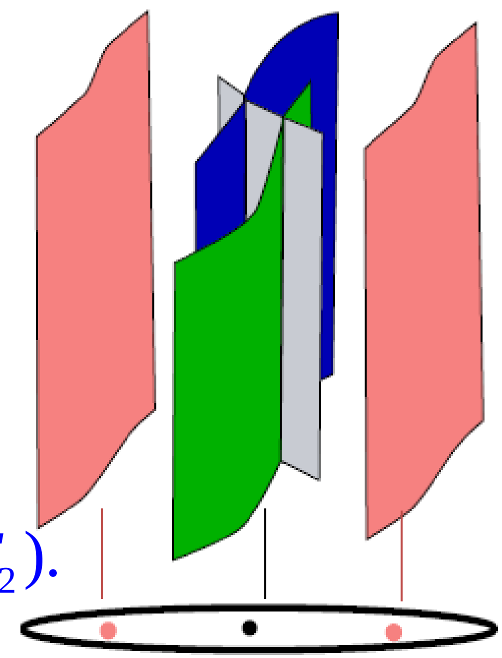


$$S_0 = \mathbb{P}^2 \cup \mathrm{Bl}^{18}(\mathbb{P}^2)$$





$$S_0 = dP_7 \cup (\mathrm{ruled}_{g=1}) \cup \mathrm{Bl}^{10}(F_2).$$



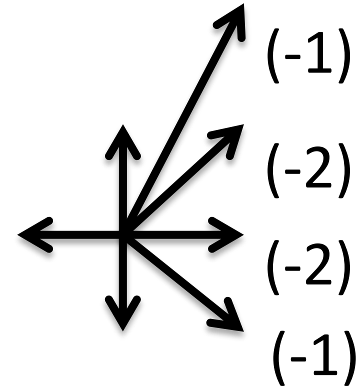
- Generalization

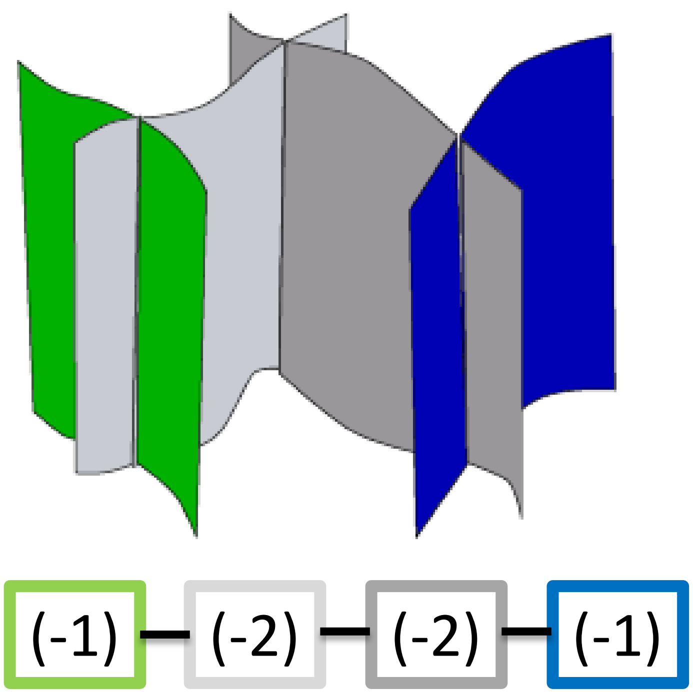
of IIA / $CY_3 = \text{ell.K3 fibr. over } \mathbb{P}^1$

with degeneration

$$= \text{ell.fibr. over } \mathrm{Bl}^k(F_n)$$

$$S_0 = \text{RES} \cup (\mathrm{T}^2 \times \mathbb{P}^1)^{k-1} \cup \text{RES}$$





Dual to Het / T2 x K3
with k NS5-branes

Ganor Hanany '96 Morrison Vafa '96

- **Generalization**

IIA / $CY_3 = \Lambda_S$ pol. K3 fibr. over $\mathbb{P}^1$ with degeneration

Type II degeneration of lattice-pol. K3 surface

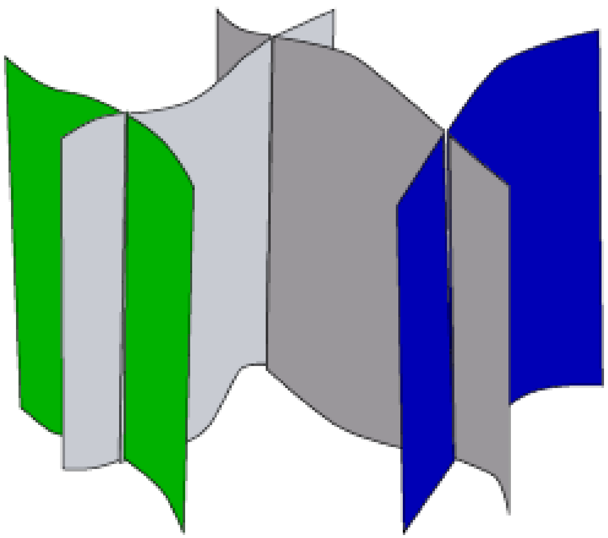
generic fibr. S_t degen. to

$$S_0 = V_0 \cup V_1 \ldots V_{k-1} \cup V_k$$

rational surfaces

$\mathbb{P}^1$-fibr over ell. curve

$$S_0 = \mathrm{RES} \cup (\mathrm{T}^2 \times \mathbb{P}^1)^{k-1} \cup \mathrm{RES}$$



(-1) — (-2) — (-2) — (-1)

Clemens—Schmid exact sequence

monodromy

$$T : \Lambda_T(S_t) \to \Lambda_T(S_t)$$

$$T =: \exp[N], \quad N^2 = 0.$$

Kulikov , Persson, Pinkham, Friedman,
Morrison, Looijenga, Saha, Scattone, ...

Type II degeneration of lattice-pol. K3 surface

generic fibr. S_t degen. to

$$S_0 = V_0 \cup V_1 \ldots V_{k-1} \cup V_k$$

rational surfaces

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$$\Lambda_T$$

$$[\text{rank } 4] \oplus R \simeq [\text{transc. lattice}]$$

Clemens—Schmid exact sequence

monodromy

$$T : \Lambda_T(S_t) \to \Lambda_T(S_t)$$

$$T =: \exp[N], \qquad N^2 = 0.$$

$$\Delta \begin{pmatrix} 1 \\ -\tau \\ \rho\tau - a^2 \\ \rho \end{pmatrix} = N \bullet \begin{pmatrix} 1 \\ -\tau \\ \rho\tau - a^2 \\ \rho \end{pmatrix}, \qquad \Delta\rho = 1.$$

Het dual: soliton,
monodromy in Narain moduli

- back to examples. (deg-2 K3 fibre)

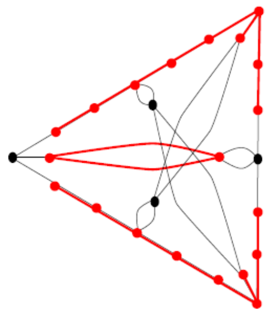
degen. to $S_0 = dP_7 \cup \mathbb{P}[\mathcal{O}_C \oplus \mathcal{L}] \cup \mathrm{Bl}^{10}(F_2)$. $\quad R = (E_7 + D_{10}); \mathbb{Z}_2,$

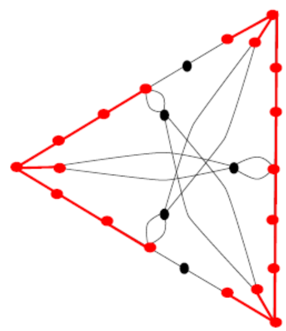
degen. to $S_0 = \mathbb{P}^2 \cup \mathrm{Bl}^{18}(\mathbb{P}^2)$ ⇒ $R = A_{17}; \mathbb{Z}_3.$

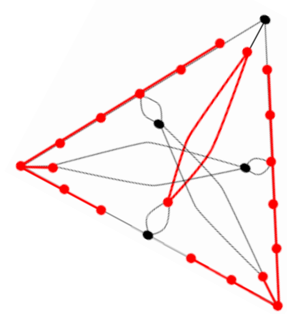
degen. to $S_0 = dP_8 \cup \mathbb{P}[\mathcal{O}_C \oplus \mathcal{L}] \cup V_2(\chi = 13)$ $\quad R = E_8^{\oplus 2} \oplus A_1$

all fall into 4 classes for deg2 K3 Type II degen. $\quad [\mathrm{rank}\ 4] \oplus R \cong \Lambda_T$

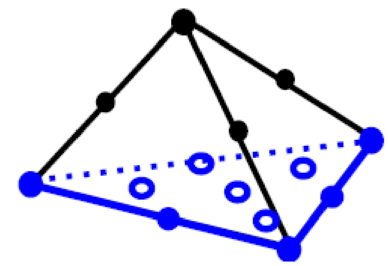
- Het interpretation: defects in $\mathbb{P}^1$ = corridor branches
  - NS 5-brane: $\Lambda_S = U,\ R = E_8 + E_8,$
  - 1st eg. above: $\Lambda_S = <+2>,\ R = (E_7 + D_{10}); \mathbb{Z}_2.$







- More varieties in degeneration of K3 surface
  - Type III: dual graph = triangulation of sphere
    - monodromy $T = \exp[N], \quad N^3 = 0.$
    - construction: Davis et.al. '13
    - more hyper-moduli -tuned solitons.



  - non semi-stable: reducible fibre with $m \neq 1.$
    - turned into semi-stable, after base change of order k
    - $T^k = \exp[N], \quad N^3 = 0.$ would-be Type II or III.

- Lattice polarization: which pair of solitons can be BPS together.