Noncommutative deformations and moduli spaces

Monday 19 November 2018 - Friday 23 November 2018

Lecture Hall(1F), Kavli IPMU

Program

Noncommutative deformations and moduli spaces
(Kavli IPMU, Nov. 19-23, 2018)
Schedule

Monday 19 November

9:30-9:50 Refreshment

9:50-10:00 Opening of the conference

10:00-11:00 Chris Brav - Nick Rozenblyum course 1

11:00-11:20 Coffee break

11:20-12:20 Yukinobu Toda

12:20-14:00 Lunch time

14:00-15:00 Mauro Porta

15:00-15:30 Photo session and tea break

15:30-16:30 Valerio Melani - Pavel Safronov course 1

16:30-16:40 Break

16:40-17:40 Yanki Lekili

Tuesday 20 November

9:30-10:00 Refreshment

10:00-11:00 Chris Brav - Nick Rozenblyum course 2

11:00-11:20 Coffee break

11:20-12:20 Atsushi Takahashi

12:20-14:00 Lunch time

14:00-15:00 Hao Wen

15:00-15:30 Tea break

15:30-16:30 Valerio Melani - Pavel Safronov course 2

16:30-16:40 break

16:40-17:40 Isamu Iwanari

Wednesday 21 November

9:30-10:00 Refreshment

10:00-10:30 Kirill Salmagambetov

10:30-11:00 Vladimir Gavran

11:00-11:15 Coffee break

11:15-11:45 Svetlana Makarova

11:45-13:15 Lunch time

13:15-14:15 Agnieszka Bodzenta (joint with MS seminar)

15:30- MS seminar: Alastair Craw

Thursday 22 November

9:30-10:00 Refreshment

10:00-11:00 Chris Brav - Nick Rozenblyum course 3

11:00-11:20 Coffee break

11:20-12:20 Sheel Ganatra

12:20-14:00 Lunch time

14:00-15:00 Andrew Macpherson

15:00-15:30 Tea break

15:30-16:30 Valerio Melani - Pavel Safronov course 3

16:30-16:40 Break

16:40-17:40 Shinnosuke Okawa

Friday 23 November

9:30-10:00 Refreshment

10:00-11:00 Chris Brav - Nick Rozenblyum course 4

11:00-11:20 Coffee break

11:20-12:20 Ludmil Katzarkov

12:20-14:00 Lunch time

14:00-15:00 Marco Roballo

15:00-15:30 Tea break

15:30-16:30 Valerio Melani - Pavel Safronov course 4

16:30-16:40 Break

16:40-17:40 Michel Van den Bergh

Joint course by Christopher Brav and Nick Rozenblyum: Derived deformation theory

We introduce the basics of derived algebraic geometry in characteristic zero and then formulate the equivalence of categories between deformation functors in this
context and differential graded Lie algebras. Examples include deformations of schemes, of objects in differential graded categories, and of differential graded categories themselves, with the latter being described by the Gerstenhaber Lie bracket on shifted Hochschild chains.

Yukinobu Toda: Birational geometry for d-critical loci and wall-crossing in Calabi-Yau 3-folds

In this talk, I will discuss birational geometry for Joyce's d-critical loci, by introducing notions such as 'd-critical flips', 'd-critical flops', etc.

I will show that several wall-crossing phenomena of moduli spaces of stable objects on Calabi-Yau 3-folds are described in terms of d-critical birational geometry, e.g. certain wall-crossing diagrams of Pandharipande-Thomas stable pair moduli spaces form a d-critical minimal model program. I will also show the existence of semi-orthogonal decompositions of the derived categories under simple d-critical flips satisfying some conditions. This is motivated by a d-critical analogue of Bondal-Orlov, Kawamata's D/K equivalence conjecture, and also gives a categorification of wall-crossing formula of Donaldson-Thomas invariants.

Mauro Porta: Tannaka duality in analytic geometry

In this talk I will survey recent work on the Tannaka duality in the analytic setting. More precisely I will describe a Tannakian reconstruction criterion for maps $X \to Y^{an}$, where X is a (derived) analytic space. I will next describe an application of this result to the Riemann-Hilbert correspondence, by generalizing the result of my paper arXiv 1703.03907 to more general coefficients. Finally, if time permits, I will sketch how combining these results with the punctured formal neighborhood technique we might hope to get an unconditional Tannakian reconstruction theorem in non-archimedean geometry. Part of this work is in collaboration with J. Holstein.

Joint course by Valerio Melani and Pavel Safronov : Introduction to shifted symplectic and Poisson geometry

The purpose of these lectures is to describe the theory of symplectic and Poisson structures on derived algebraic stacks, with particular emphasis on examples coming from the moduli of bundles and Poisson-Lie groups. We will start by introducing derived symplectic geometry, giving the main definitions of symplectic and Lagrangian structures, together with important existence results. Even if building a theory of Poisson structures is classically not harder than building a theory of symplectic structures, the situation in derived algebraic geometry is completely different. We will explain the intrinsic problems preventing the naive definition of Poisson structure to work in the derived setting. The solution is then given by formal localization, a highly non-trivial procedure which we will try to explain during the lectures. We will then mention comparison results between derived symplectic and Poisson geometry. If time permits, we will explain an application of the theory leading to deformation quantization of derived moduli spaces.

Yanki Lekili: Homological mirror symmetry for higher dimensional pants

We prove that the partially wrapped Fukaya category of the complement of (n+2)-generic hyperplanes in CP^n (n-dimensional pants) with respect to certain stops is equivalent to a certain categorical resolution of the derived category of the singular affine variety x_1x_2..x_{n+1}=0. By localizing, we deduce that the (fully) wrapped Fukaya category of n-dimensional pants is equivalent to the derived category of x_1x_2...x_{n+1}=0. This is joint work with A. Polishchuk.

Atsushi Takahashi : Primitive forms and noncommutative deformation

An overview of Kyoji Saito’s theory of primitive forms is given.
In particular, we will see there the prototype of noncommutative Hodge structure of Calabi-Yau type. If there is time, we will discuss future issues on “noncommutative universal unfoldings”, noncommutative deformations associated to Landau-Ginzburg orbifolds.

Hao Wen (YMSC, Tsinghua University): Landau-Ginzburg model via L^2 Hodge theory

Let X be a non-compact Calabi-Yau manifold and f be a holomorphic function on X with compact critical locus, satisfying a general asymptotic condition. We construct a suitable subspace of smooth polyvector fields on X which carries a dGBV structure with a trace map and satisfies the Hodge-to-de Rham degeneration property. This construction is based on the establishment of a version of L^2 Hodge theory on X, which puts Landau-Ginzburg B-model of the pair (X,f) into the same setting as compact Calabi-yau manifolds. It leads to a Frobenius manifold by the Barannikov-Kontsevich construction and can be viewed as a generalization of Kyoji Saito's higher residue theory and primitive forms for isolated singularities. This is a joint work with Si Li.

Isamu Iwanari (Tohoku university): Calculus of infinity-categories

For a smooth manifold or an algebraic variety, its multivector fields and differential forms admit several algebraic operations which have been fundamental in geometry. As for a noncommutative associative algebra, an analogous structure appears on the pair of its Hochschild cochain and Hochschild chain. This algebraic structure is encoded by the so-called calculus/Kontsevich-Soibelman operad. I would like to describe a construction of an algebraic structure on the pair of Hochschild invariants of a stable infinity-category.

Kirill Salmagambetov: Traces, Hochshild homology and derived free loop space

Traces in symmetric monoidal categories is a categorical formalism generalizing the familiar notion of trace from linear algebra. Many important constructions in derived and noncommutative algebraic geometry involve taking traces in a suitable symmetric monoidal category. Examples include free loop space of stack and Hochshild homology of large dg categories. I will give a friendly introduction to traces in symmetric monoidal categories, their functoriality and explain why traces of automorphisms come equiped with a functorial action of the circle group. The resulting formalism turns out to be useful for different comparison theorems. As an example, we will consider Dg category of quasicoherent sheaves on a stack X and see how the canonical S^1 - action on it's Hochshild homology can be interpreted in terms of derived geometry of the free loop space of X.

Vladimir Gavran: Introduction to non-commutative Calabi-Yau geometry

In my talk I will give the definition of Calabi-Yau structure on a smooth DG-category and describe how this notion provides a noncommutative analog of Calabi-Yau varieties.

Svetlana Makarova: Combinatorial constructions of derived autoequivalences of grassmannians

In this talk, I will review a recent work on "magic windows" by several groups of people and consider an explicit example. A new (but more complicated) proof of fullness of Kapranov's collection on grassmannians will follow.

Agnieszka Bodzenta: Categorified non-commutative deformations and abelian envelopes

I will discuss categorified non-commutative deformations of a finite collection of objects in an abelian category. The deformation functor is a non-commutative Deligne-Mumford stack. To such a stack we assign a monad and a comonad. I will prove that the Eilenberg-Moore categories of these provide abelian envelopes for the exact subcategory of the original abelian category generated by the deformed objects.

Sheel Ganatra: Structural results for wrapped Fukaya categories

I will describe some new structural results for wrapped and partially wrapped Fukaya categories of Stein manifolds, emphasizing relationships to structures in noncommutative geometry, mirror symmetry, and sheaf theory. This is joint work with J. Pardon and V. Shende.

Andrew Macpherson: A Yoneda philosophy of correspondences

Cohomology is bivariant, which means that to a morphism f it associates not only a pullback map f^*, but also (under certain conditions) an Umkehr map in the opposite direction. These maps satisfy a "push-pull" or "base change" identity. Everyone knows that this implies that cohomology can be thought of as a functor out of a certain category CORR of "correspondences", whose morphisms are "rooves" and whose composition law is defined by taking a fibre product of kernels. This fact is a crucial ingredient in the construction of cohomological 2D field theories.

In higher category theory, specifying objects by describing the morphism spaces and composition law explicitly --- as we just did with correspondences --- is rather inconvenient. Rather, it is better to define things via their universal properties. In this talk, I will give a universal interpretation for CORR in terms of "bivariant functors" into an (∞,2)-category, which takes out the pain from constructing functors out of CORR.

Shinnosuke Okawa : Noncommutative del Pezzo surfaces as AS-regular I-algebras

Abstract: Noncommutative projective planes and noncommutative quadrics are defined as abelian categories associated to the so-called 3-dimensional AS-regular quadratic (resp. cubic) Z-algebras. Moreover there is a bijective correspondence between such algebras and certain geometric data consisting of a genus one curve and a collection of line bundles on it. I will talk on a work in progress with Tarig Abdelgadir and Kazushi Ueda which aims to generalize this story to obtain solid definition and classification of noncommutative del Pezzo surfaces of all other types as well.

Ludmil Katzarkov: Categorical Curve Complexes

In this talk, we will introduce a new categorical construction.
Applications will be considered.

Marco Robalo: Matrix Factorisations and Motivic Vanishing Cycles

In this talk I will explain a result obtained in joint work with A. Blanc, B. Toen and G. Vezzosi. comparing the motive associated to the dg-category of matrix factorisations of an LG-pair, and the construction of motivic vanishing cycles of the pair.

Michel Van den Bergh : A k-linear triangulated category without a model.

We give an example of a triangulated category, linear over a field of characteristic zero, which does not carry a DG enhancement. To this end we develop the theory of pretriangulated A_n categories and we construct a non-trivial example of such a category by gluing two pre-triangulated A_infty categories over an A_n functor which cannot be extended to an A_infty functor. The only previous examples of triangulated categories without a model have been constructed by Muro, Schwede and Strickland. These examples are however not linear over a field. This is joint work with Alice Rizzardo.