Anomaly polynomial of general 6d SCFTs

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Review: What are ’t Hooft anomalies?

- ’t Hooft anomaly: obstruction to gauging global symmetries.
  
  Couple the theory to background gauge fields $A_\mu, g_{\mu\nu}$
  
  $\rightarrow$ effective action fails to be gauge invariant:

  \[
  \delta W_d[g_{\mu\nu}, A_\mu] = \int I_d^{(1)}[g_{\mu\nu}, A_\mu].
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- Descent equation and anomaly polynomial:
  $$I_{d+2} = dI_{d+1}^{(0)} , \; \delta I_{d+1}^{(0)} = dI_d^{(1)}.$$ 

  Anomaly polynomial $I_{d+2}$: polynomial of characteristic classes
  $$p_1(T) = -\frac{1}{8\pi^2} \text{tr} \; R^2, \; p_2(T) = \frac{1}{128\pi^2} \left((\text{tr} \; R^2)^2 - 2 \text{tr} \; R^4 \right) \text{ etc...}$$
Anomalies and 6d SCFTs

- 6d SCFT: self-dual 2-form gauge fields and tensionless strings.

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- Cancelation of gauge anomalies: strong constraint for 6d SCFTs.
  “Atomic classification” [Bhardwaj ’15] [Heckman, Morrison, Rudelius, Vafa ’15]
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- ’t Hooft anomalies for global symmetries:
  
  - $N^3$ scaling law of d.o.fs
  
  - Central charges of compactified theory [Benini, Tachikawa, Wecht ’09]
  
  - RG flow between 6d SCFTs [Heckman Morrison Rudelius Vafa ’15]

  etc...
How to calculate ’t Hooft anomalies of 6d SCFTs?

- Gravitational calculation:
  
  embed 6d SCFT into M-theory and use anomaly inflow.

  [(Freed,) Harvey, Minasian, Moore ’98] [Yi ’01] [Ohmori, HS, Tachikawa ’14]

Field theory calculation:
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- **Field theory calculation:**
  
  deform 6d SCFT to free field theory and use anomaly matching.

  [Ohmori, HS, Tachikawa, Yonekura '14] [Intriligator '14]
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In this short talk, I will explain the field theoretical calculation of anomaly polynomial of 6d $\mathcal{N} = (2, 0)$ theory.
Anomaly polynomial of 6d $\mathcal{N}=(2,0)$ theory

- $G$-type $\mathcal{N}=(2,0)$ theory: IIB on ADE orbifold $\mathbb{C}^2/\Gamma_G$.

Anomaly polynomial conjecture: [Intriligator '00]

\[
I_{G}^{\mathcal{N}=(2,0)} = \frac{h_{G}^{\vee}d_{G}}{24}p_2(N) + r_{G}I_{\mathcal{N}=(2,0)\text{ tensor}},
\]

\[
I_{\mathcal{N}=(2,0)\text{ tensor}} = \frac{1}{48} \left( p_2(N) - p_2(T) + \frac{1}{4}(p_1(T) - p_1(N))^2 \right).
\]

\[
h_{\text{SU}(k)}^{\vee}d_{\text{SU}(k)} = k^3 - k \quad \text{N: SO}(5) \text{ R-symmetry}
\]
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$$h^\vee_{SU(k)} d_{SU(k)} = k^3 - k \quad N: SO(5) \text{ R-symmetry}$$

- When $G = A_{k-1}, D_k$, gravitational calculation is available:
  
  $G = A_{n-1} \rightarrow$ coincident $k$ M5-branes, [(Freed, ) Harvey, Minasian, Moore ’98]
  
  $G = D_n \rightarrow$ coincident $k$ M5-branes + orientifold. [Yi ’01]
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- Field theory calculation is valid for any $G$. [Ohmori, HS, Tachikawa, Yonekura]
Tensor branch RG flow

- Tensor branch RG flow: giving tension to self-dual strings.

\[ G\text{-type } \mathcal{N}=(2,0) \text{ theory } \rightarrow r_G \text{ free } \mathcal{N} = (2,0) \text{ tensor multiplets.} \]

UV \hspace{1cm} IR \hspace{1cm} \begin{cases} B_2, \phi^i=1\ldots5, \text{ fermions} \end{cases}

We choose to preserve R-symmetry \( SO(4)_R \subset SO(5)_R. \)
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We choose to preserve R-symmetry \( \text{SO}(4)_R \subset \text{SO}(5)_R \).

Anomaly matching on tensor branch:

\[ I_G^{\mathcal{N}=(2,0)} = (\text{anomaly matching term}) + r_G I_{\mathcal{N}=(2,0)}^{\text{tensor}}. \]

Origin of anomaly matching term?
Green-Schwarz mechanism for anomaly matching

- Integrate out massive strings: induce electric/magnetic coupling for $B_2$

$$\Delta L = \int_{X_6} \Omega^{ij} B_i I_j$$ and $dH_i = I_i$ $i = 1 \cdots r_G$.

Bianchi identity
Green-Schwarz mechanism for anomaly matching

- Integrate out massive strings: induce electric/magnetic coupling for $B_2$

  \[ \Delta L = \int_{X_6} \Omega^{ij} B_i I_j \quad \text{and} \quad dH_i = I_i \quad i = 1 \cdots r_G. \]

- Additional contribution from Green-Schwarz mechanism:

  \[ I^{\text{GS}} = \frac{1}{2} \Omega^{ij} I_i I_j \]

  \[ I^{\mathcal{N}=(2,0)}_G = I^{\text{GS}} + r_G I^{\mathcal{N}=(2,0)} \text{ tensor}. \]

How to determine $I_i$?
$S^1$ compactification of 6d $\mathcal{N}=(2,0)$ theory

- $G$-type 6d $\mathcal{N}=(2,0)$ theory on $S^1 \rightarrow 5d \ G \ \mathcal{N}=2$ SYM.

  6d tensor branch $\rightarrow$ 5d Coulomb branch: $G \rightarrow U(1)^{r_G}$.

  $\langle \phi^a_{i=1\ldots4} \rangle = 0$ and $\langle \phi^a_5 \rangle = v^a$, then $SO(5)_R \rightarrow SO(4)_R$. 

- Massive strings
  - Massive $\mathcal{N}=2$ vectors (roots of $G$) with mass $v$.
  - Integrating out:
    - Induced Chern-Simons terms $S_{CS} = \Omega_{ij} A^i_I A^j_J$; $A^i_I$: $U(1)^i_g$ gauge field.
  - Reduce to ordinary 1-loop calculation!
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- Massive strings $\to$ massive $\mathcal{N}=2$ vectors $\Phi_\alpha$ ($\alpha$: roots of $G$) w/ mass $v \cdot \alpha$.

  Integrating out $\Phi_\alpha$: induced Chern-Simons terms

  $$S^{CS} = \Omega^{ij} A_i A_j, \quad A_i : U(1)_i \text{ gauge field}.$$ 

  Reduce to ordinary 1-loop calculation!
Induced Chern-Simons term for $A_i \ i = 1 \cdots r_G$:

\[
\frac{1}{2} \sum_{\alpha > 0} (\alpha \cdot A) \left[ (c_2(L) + \frac{2}{24} p_1(T)) - (c_2(R) + \frac{2}{24} p_1(T)) \right]
\]

\[= \rho \cdot A(c_2(L) - c_2(R)),\]

\[\rho = \frac{1}{2} \sum_{\alpha > 0} \alpha: \text{Weyl vector}, \quad \text{SO}(4)_R \sim \text{SU}(2)_L \times \text{SU}(2)_R.\]
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Green-Schwarz contribution:

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\frac{1}{2} \langle \rho, \rho \rangle (c_2(L) - c_2(R))^2 = \frac{h_G^\vee d_G}{24} (c_2(L) - c_2(R))^2.
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pos real mass fermions  \quad neg real mass fermions

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Anomaly polynomial of $G$-type $\mathcal{N}=(2,0)$ theory:

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I_G^{\mathcal{N}=(2,0)} = \frac{h_G^\vee d_G}{24} p_2(N) + r_G I^{\mathcal{N}=(2,0)} \text{ tensor}.
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Conclusions

- Anomaly polynomial: exactly computable quantity of 6d SCFTs.
- We established the field theoretical way to calculate them.
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- Anomaly matching on tensor branch:
  Need massless spectrum/Green-Schwarz coupling on tensor branch.
- **F-theory geometry**: determine both massless spectrum and Green-Schwarz coupling \(\rightarrow\) **anomaly polynomial of general 6d SCFTs.**
Conclusions

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- We established the field theoretical way to calculate them.

- Anomaly matching on tensor branch:
  Need massless spectrum/Green-Schwarz coupling on tensor branch.

- **F-theory geometry**: determine both massless spectrum and Green-Schwarz coupling $\rightarrow$ anomaly polynomial of general 6d SCFTs.

- These polynomials may be used to prove a-theorem for 6d SCFTs/investigate compactification etc......