Direct Neutrino Mass Measurements
Using Weak Decays

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Outline

1. Motivation
2. Direct $\nu$-mass measurement principle ($\beta$-decay)
3. History of direct $\nu$-mass results
4. Introduction to the KATRIN experiment

Fermi (1933)
Motivation

- Oscillation experiments have unambiguously established $m_\nu > 0$ eV
  - Nobel prize in 2015
- Non-zero neutrino masses are not easily explainable within the SM
  - Hint towards lepton number violation, new source of CP violation, sterile (right-handed) neutrinos, new energy scale, ...
- $m_\nu$ of major interest for astrophysics and cosmology

Gonzalez-Garcia, Maltoni, Schwetz (2014)
Neutrino mass spectrum

- Flavor states ≠ propagation (mass) states
- Oscillation results: tiny mass splittings + large mixing

![Diagram showing Neutrino mass spectrum]

Absolute ν mass scale?
Ways to the absolute neutrino mass scale

- Cosmology
  - CMB, LSS, BAO
  - model-dependent
  - $\sum_i m_i \lesssim 0.12 - 1.0$ eV

- Neutrinoless double beta decay ($0\nu\beta\beta$)
  - model-dependent (Majorana CP phases)
  - $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i| < 0.2 - 0.4$ eV
  - GERDA, EXO/nEXO, SNO+, MAJORANA, CUORE, CANDLES, NEXT, KamLAND-Zen, ...
Direct neutrino mass search

- SN1987a (time-of-flight analysis)
  - \( m(\nu_e) < 5.7 \text{ eV (95\% C.L.)} \)

- Kinematics / spectroscopy of weak decays
  1. \( \beta \)-decay of tritium (\( T_2 \))
     - KATRIN, PROJECT 8
  2. electron capture of holmium (\(^{163}\text{Ho}\))
     - ECHO, HOLMES
  - model-independent:
    \[
    m^2(\nu_e) = \sum_i |U_{ei}^2| m_i^2
    \]

  - status: \( m(\nu_e) < 2000 \text{ meV (95\% C.L.)} \)
  - potential: \( m(\nu_e) < 200 \text{ meV (90\% C.L.)} \)
Measurement Principle
of tritium $\beta$-decay spectroscopy
Shape of the $\beta$ electron spectrum

- $\beta$-decay of tritium: $^3H \rightarrow ^3He^+ + \bar{\nu}_e + e^-$
- spectroscopy of the emitted electron kinetic energy

\[
\frac{d\Gamma}{dE} \propto (E_0 - E) \cdot \sqrt{(E_0 - E)^2 - m_{\nu_e}^2} \cdot \theta(E_0 - E - m_{\nu_e}^2)
\]

\[
m^2(\nu_e) = \sum_i |U_{ei}|^2 m_i^2
\]

\[
\frac{d\Gamma}{dE}
\]

fraction of $\beta$ electrons in last eV before $E_0$:
$2 \cdot 10^{-13}$
Standard experimental tritium $\beta$-decay setup

- essentially since 1952 (only few exceptions)

- **$\beta$ emitter**
  - tritium source

- **electron collimation**
  - magnetic transport
  - high luminosity
  - gaseous or solid
  - good understanding of energy loss processes

- **energy filter**
  - spectrometer
  - adiabatic / undisturbed transport of $\beta$ electrons
  - removal of residual $T_2$
  - high energy resolution
  - low background

- **counter**
  - detector
  - high efficiency
  - low backscattering
  - low background
Response of the experimental apparatus

- Response function $R(E)$ describes the influence of the apparatus on the $\beta$-decay spectrum
  - Spectrometer energy resolution / transmission characteristics
  - Energy loss processes of $\beta$ electrons
Observed signal

- For magnetic spectrometers (till early 90s)
  \[ \dot{N}_{\text{sig}}(E) \propto R(E) \otimes \frac{d\Gamma}{dE}(m_{\nu e}^2, E_0) \]

- For electrostatic spectrometers (Troitsk, Mainz, KATRIN)
  \[ \dot{N}_{\text{sig}}(U) \propto \int_{qU}^{\infty} R(U, E) \cdot \frac{d\Gamma}{dE}(m_{\nu e}^2, E_0) \, dE \]

Kawakami et al. (1990), Tokyo
Standard 4 parameter model fit

- Number of observed events $N_i$ at retarding step $i$ with retarding voltage $U_i$ (recent setups):

$$N_i \propto t_i \cdot \left[ A_{\text{sig}} \cdot \int_{qU_i}^{\infty} R(U_i) \cdot \frac{d\Gamma}{dE}(m_{\nu_e}^2, E_0) \, dE + R_{\text{bg}} \right]$$

- Parameter of interest:
  - Squared neutrino mass $m_{\nu_e}^2$

- Nuisance parameters:
  - Tritium endpoint energy $E_0$
  - Signal amplitude $A_{\text{sig}}$
  - Mean background rate $R_{\text{bg}}$

- $A_{\text{sig}}, E_0, R_{\text{bg}}$ are correlated with $m_{\nu_e}^2$ and apriori not known well enough
Hypothetical neutrino mass signal (KATRIN)
Data analysis – probabilistic model

• Having several data points $N_{\text{obs},i}$
  - number of counted / observed electrons
  - random variable with Poissonian distribution (radioactive decays)
  - with statistical error $\sigma = \sqrt{N_{\text{mod},i}}$
  - no binning required

• and a model prediction for the mean value $N_{\text{mod},i}(m_v^2, A_{\text{sig}}, E_0, R_{\text{bg}})$

• formulate likelihood

$$L = \prod_{i} p \left( N_{\text{obs},i} \mid N_{\text{mod},i} \right)$$

• and $\chi^2$ statistic for $p \approx$ Gaussian distr. with $\sigma = \sqrt{N}$

$$-2 \log L = \chi^2 = \sum_{i} \left( \frac{N_{\text{obs},i} - N_{\text{mod},i}}{\sigma} \right)^2$$
Data analysis – parameter inference

• Method of least squares (most common):
  • $\chi^2$ minimization $\rightarrow \hat{m}_\nu^2$ best-fit estimate
  • determine statistical error $\sigma_{\text{stat}}(m_\nu^2)$ from profile likelihood / $\Delta \chi^2$ shape
    ($\Delta \chi^2 = 1$ rule, considering nuisance parameters)
• or ensemble tests

Kraus et al. (2005) Mainz $\chi^2$ curves

KATRIN ensemble tests for $m_\nu^2 = 0$ eV$^2$
Data analysis – systematic errors

• Common practice for systematic errors:
  • Evaluate shift of the estimate $\Delta \hat{m}^2$ for each systematic effect / variable + covariances using Monte Carlos

  $$\sigma_{\text{syst}}(m^2) = \sqrt{\Delta_1^2 + \Delta_2^2 + \rho(\Delta_1\Delta_2) + \cdots}$$

• Add systematic errors in quadrature, incl. correlation terms:

  $$\sigma(m^2) = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2}$$

• Combine statistical and systematic error in quadrature:

  • Publish distinct results if a systematic is 'not parameterizable'

    • E.g. different final state calculations

Fritschi et al. (1986) Zürich
Data analysis – Bayesian approach

- rarely used in direct ν-mass analyses
- perform Markov Chain Monte Carlos to sample posterior probability density distributions from the likelihood based on Bayes’ Theorem
- no distinction between nuisance and systematic parameters necessary
  - likelihood constraints for each systematic (e.g. Gaussian prior)
- priors must be agreed on (especially $m_\nu$ or $m_\nu^2$ resp.)
Why measure the *squared* neutrino mass?

- \( \frac{d\Gamma}{dE} \propto \sqrt{(E_0 - E)^2 - m_\nu^2} \), for \( m_\nu^2 \ll E \): \( \chi^2 \propto (m_\nu^2)^2 \)
- \( \chi^2 \) is parabolic in \( m_\nu^2 \)
- \( m_\nu^2 \) estimates are Gaussian distributed – very convenient error treatment
- Errors for \( m_\nu \) strongly depend on absolute value of \( m_\nu \)
  \rightarrow for \( m_\nu^2 \) they don’t.
Continuation for non-physical $\hat{m}_ν^2 < 0$

- Required to reflect statistical fluctuations of observed data
  - $χ^2$ statistic / goodness-of-fit
  - Frequentist confidence interval construction

- Demands arbitrary non-physical continuation of the model
  - Symmetrical continuous likelihood / $χ^2$ around minimum

- Not required in bayesian methods
  - Choose prior with $m_ν^2 ≥ 0$.

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**Distribution of $\hat{m}_ν^2$ estimates without continuation**

**$β$ decay rate $dΓ/dE$ in $s^{-1}eV^{-1}$**

- $m_ν^2 = 1.0$ eV$^2$
- $m_ν^2 = 0.0$ eV$^2$
- $m_ν^2 = -1.0$ eV$^2$, tachyonic
- $m_ν^2 = -1.0$ eV$^2$, extrapolated

**Electron energy $E - E_0$ in eV**

- $0 < \hat{m}_ν^2 < 0.05$
- $-2 < E - E_0 < 1$
Results of Previous $\beta$-Decay Experiments
and the statistical method involved
First tritium $\beta$-decay $\nu$-mass limit

- Curran et al., Phys. Rev. 76 (1949)
- pulse height distribution measured with proportional counter
- $0 \text{ keV} < m(\nu_e) < 1 \text{ keV}$
Direct $\nu$-mass measurements since 1990

- about a dozen direct $\nu$-mass experiments since 1949
- early estimates clearly tend towards non-physical values
Interpretation of non-physical $m^2_\nu$ estimates

A. k. a. the *neutrino problem* or *bounded Gaussian mean* problem:

- Many experiments with $m^2_\nu$ best-fit in a non-physical region
- Resulting in small or empty classical *confidence intervals*

Bob Cousins, Virtual Talk (2012)
Interpretation of non-physical $m^2_\nu$ estimates

- Some workarounds do not produce correct Frequentist coverage

upper limit = $\max(x, 0) + 1.64\sigma$

flip-flopping

overcoverage

undercoverage

Bob Cousins, Virtual Talk (2012)
Interpretation of non-physical $m^2_\nu$ estimates

- In 1979 the particle data group (PDG) recommended a *Bayesian approach* with flat prior in $m^2_\nu \geq 0$ (curve b):

Robertson & Knapp (1988)
Interpretation of non-physical $m^2_\nu$ estimates

• In 1997 Feldman & Cousins present their unified approach
  • Neyman confidence belt construction with ordering principle
  • Non-null Frequentist confidence intervals beyond physical boundaries
  • Correct global coverage

• Recommended by particle data group (PDG) since 1998
Comparison of confidence / credible intervals

95% C.L. intervals

μ = -2σ

μ = 0

μ = +2σ
Critical systematic effects

- Underestimated / missed energy loss processes usually lead to smaller (or negative) $m_\nu^2$
- Usually, these systematic errors increase for smaller energies (further away from $E_0$)
Critical systematic effects

- Any missed experimental broadening of the energy spectrum with Gaussian width $\sigma$ leads to

$$\Delta m^2 \approx -2\sigma^2$$

- Thermal Doppler broadening (source)
- High voltage fluctuations (spectrometer)

- Often without any effect on the $\chi^2$ statistic (goodness-of-fit) of a measurement!
The **Karlsruhe TRItium Neutrino Experiment**

A next current generation effort of probing the $\nu$ mass scale
KATRIN’s primary physics objective

- Improve sensitivity on $m(\nu_e)$ by one order of magnitude to 200 meV (90% C.L.)

- Statistical and systematic errors on $m^2(\nu_e)$ must be improved by two orders of magnitude to $\sigma_{\text{stat}}(m^2_\nu) \approx \sigma_{\text{syst}}(m^2_\nu) \approx 0.017 \text{ eV}^2$
KATRIN setup

Source & transport section
- Windowless gaseous tritium source
- Intensity \(10^{11} \text{ s}^{-1}\)
- Stability \(10^{-3} \text{ h}^{-1}\)
- Isotopic purity (> 95%)
- Tritium retention factor (> \(10^{14}\))
- Adiabatic transport of electrons

Spectrometer & detector section
- Spectrometer UHV \((p < 10^{-11} \text{ mbar})\)
- Energy resolution (<1 eV at 18.6 keV)
- High voltage stability (ppm/month)
- Low background rate (10 mcps)
- High detection efficiency (mcps to kcps)
Design aspects

$^3\text{H} \beta$-decay

- Short $T_{1/2}$ of 12.3 y → high-intensity source
- Low endpoint of 18.6 keV → good rel. signal strength
- Gas, closed loop → high isotopic purity
- Computation of final states, radiative & recoil corrections

MAC-E filter technique

Magnetic Adiabatic Collimation with Electrostatic filter
Picard et al., NIM B63 (1992) 345

Isotropic emission, strong $B_s$
Energy filtering, weak $B_{\text{min}}$
Resolution:

$$\frac{\Delta E}{E} = \frac{B_{\text{min}}}{B_{\text{max}}} = \frac{1}{20000} \quad \text{(at KATRIN)}$$

$\mu = \frac{E_\perp}{B} = \text{const.}$
KATRIN measuring strategy

- Choice of measuring time distribution determines balance between statistical and systematic errors
- Absolute observed rates, thus statistical errors, are smaller at lower spectrum energies
- Systematic uncertainties (energy loss) are larger at lower spectrum energies
KATRIN sensitivity

corresponds to 5 calender years of data-taking
v-mass ... and more!

Explore physics potential
• close to the spectral endpoint $E_0$:

RH currents
Bonn et al. (2011)

Violation of Lorentz symmetry
e.g. Diaz, Kostelecky & Lehnert (2013)

Constraining local CvB overdensities
e.g. Kaboth & Formaggio (2010)

Search for eV-scale sterile $\nu$

• and further away from $E_0$:
Search for keV-mass scale sterile $\nu$ as WDM candidates
S. Mertens et al. (2015)

standard operation mode for KATRIN

non-standard operation, requires novel concepts
Status

• All hardware components are on site
  • Beam line integration and commissioning ongoing

• Analysis strategy and software in final dev. stage
  • Comprehensive probabilistic model
  • Gas dynamics simulations of the tritium source
  • Response function modelling incl. energy loss processes
  • 3D particle tracking suite
  • Blinding strategies evaluated
  • Frequentist (unified approach) & Bayesian (Markov Chain MCs) methods implemented

• First tritium runs in 2017
Thank you for your attention.
Backup slides
Mainz experiment results

- First analysis in 1992: $m^2(\nu_e) = -39 \pm 34_{\text{stat}} \pm 15_{\text{syst}} \text{eV}^2$

- A source-related systematic mistake was discovered later on
  - film of molecular tritium $T_2$ quench-condensed onto substrate
  - temperature-activated roughening, **missed increase of inelastic scattering**

- Final analysis in 2005: $m^2(\nu_e) = -0.6 \pm 2.2_{\text{stat}} \pm 2.1_{\text{syst}} \text{eV}^2$
  - $m(\nu_e) < 2.3 \text{ eV} \ (90\% \ C.L.)$
sterile $m_\nu$ sensitivity

- standard measuring time distr.
  optimized for active $\nu$ search
- 5 cal. years of data taking
- active $\nu$ mass is fixed at 0 eV

reactor anomaly
combined fit 90% C.L.
K. N. Abazajian et al. 2012

exclusion curve confidence levels

- KATRIN 5$\sigma$
- KATRIN 4$\sigma$
- KATRIN 3$\sigma$
- KATRIN 95%
- KATRIN 90%
- KATRIN 68%
Principle of electron capture (EC) spectroscopy

- Electron capture on Holmium $^{163}$Ho:
  - $e^- + ^{163}_{67}Ho \rightarrow ^{163}_{66}Dy^* + \nu_e$
- Deexcitation energy of daughter atom in form of electrons and x-rays recorded in microcalorimeters
- Shape distortion close to the Q-value depends on $m_{\nu e}^2$
Theoretical corrections

- Radiative corrections & recoil effects are well understood
- Final states of the excited daughter isotopes \( ^3\text{HeT}^+ \), \( ^3\text{HeD}^+ \), \( ^3\text{HeH}^+ \) lead to a broadening of the spectrum

Excited final state distribution of \( T_2 / DT \)
- 0.1 eV binning
- \( T = 30 \) K

\[ \beta \text{ spectrum incl. final states and corrections} \]
Why tritium?

\[ \frac{d\Gamma}{dE} \propto |M|^2 \cdot F(Z, E) \cdot p(E + m) \cdot (E_0 - E) \cdot \sqrt{(E_0 - E)^2 - m_{\nu e}^2} \]

- **Superallowed transition:**
  - Matrix element M is not energy dependent

- **Low endpoint energy:**
  - fraction of decays at the endpoint is comparatively high \( \sim \frac{1}{E_0^3} \)

- **Short half life:**
  - specific activity is high
  - low amount of source material / less inelastic scattering

- **Hydrogen isotope:**
  - simple atomic shell
  - final states precisely calculable