Today's Talk

Peter Arnold

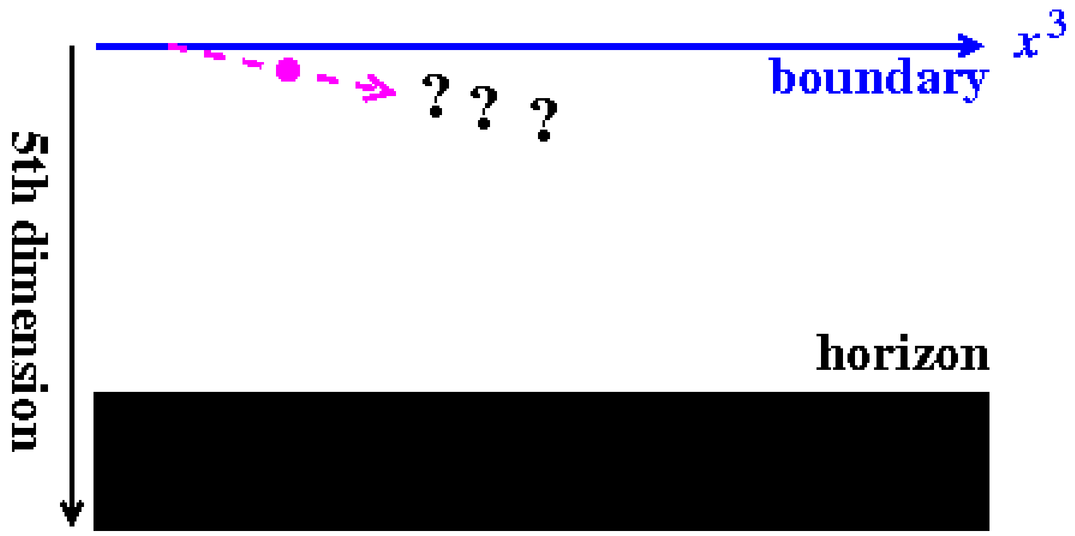
I. A fun problem in gravity

II. What it's good for

arXiv: 1212.3321
with Philip Szepietowski, Diana Vaman, and Gabriel Wong

A fun problem in gravity

Suppose a graviton is launched from the boundary of (AdS$_5$-Schwarzschild) $\times$ S^5.
What happens to it?



A fun problem in gravity

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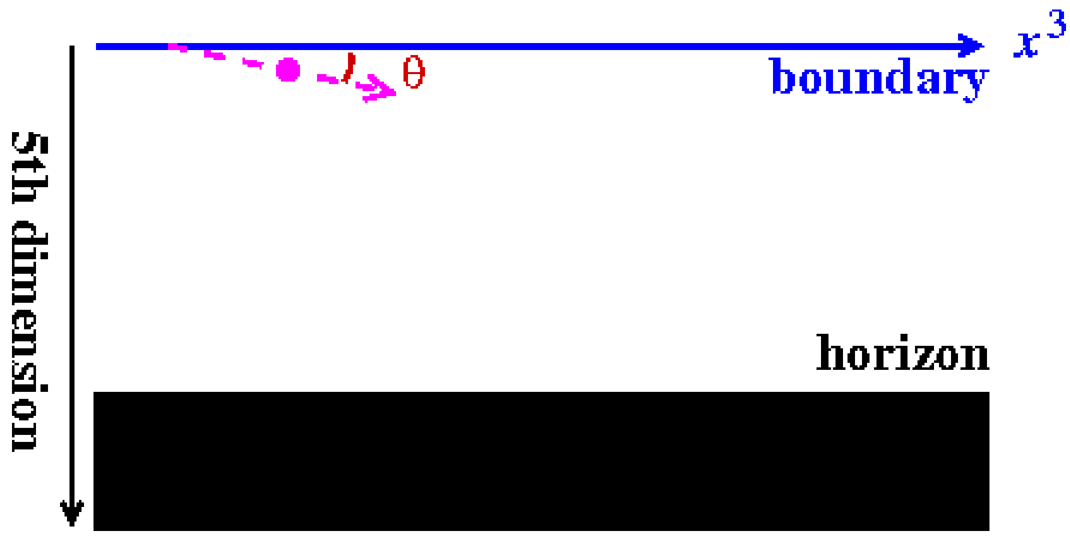
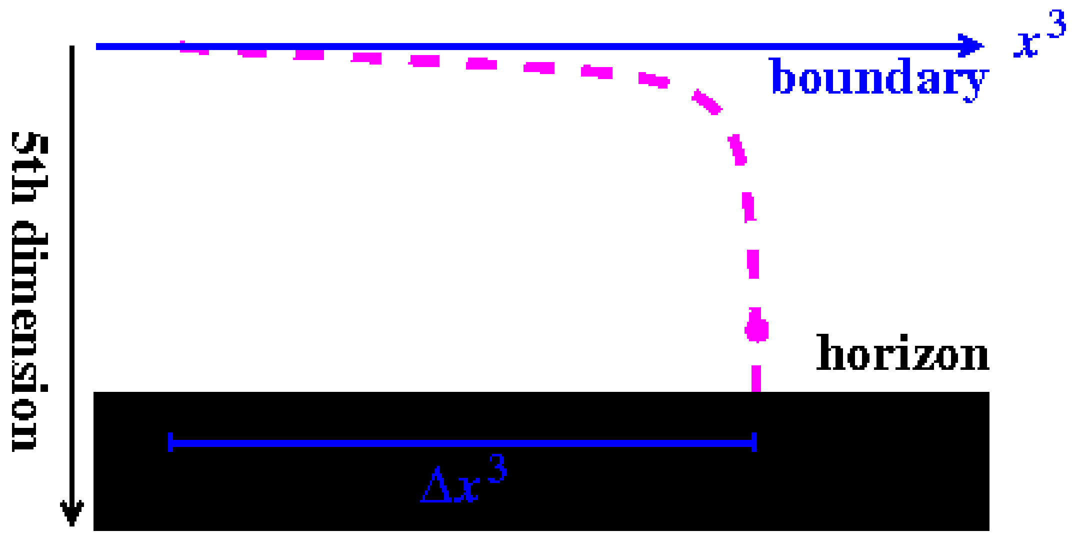


I'm going to be interested in gravitons (or photons or whatever)

- that start out moving at a small angle θ relative to the boundary
- with large momentum q_3 in the x^3 direction
- with a localized wave function in AdS_5-Schwarzschild

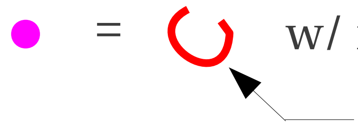
First answer



Note for later:

- graviton travels a finite distance Δx^3 in x^3
- takes infinite boundary-time x^0 to fall into black hole

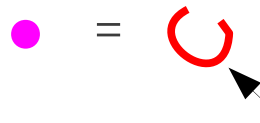
But really



w/ internal degrees of freedom in ground state

proper size ~ (string tension)$^{-1/2}$ ~ $(\alpha')^{1/2}$

But really



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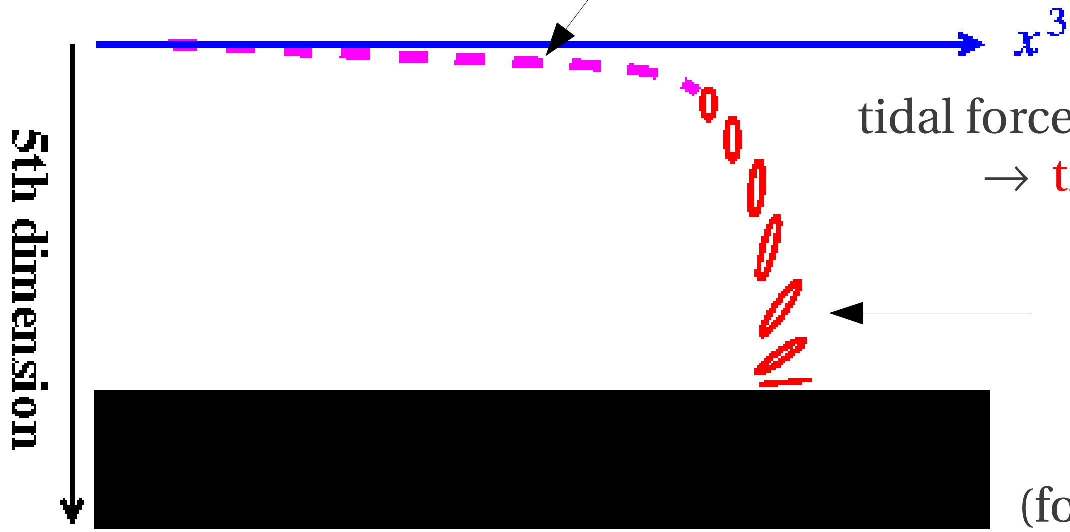
graviton is a quantum closed string



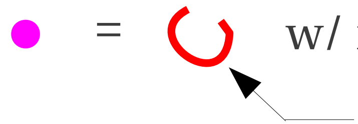
tidal forces eventually exceed string tension
→ tidal stretching of string!

now a classical closed string

(for large enough momentum q_3)
(also for $g_{\text{string}} \to 0$, else string can break)

Q: Is it possible to quantitatively calculate the late-time probability distribution of classical string configurations?

But really



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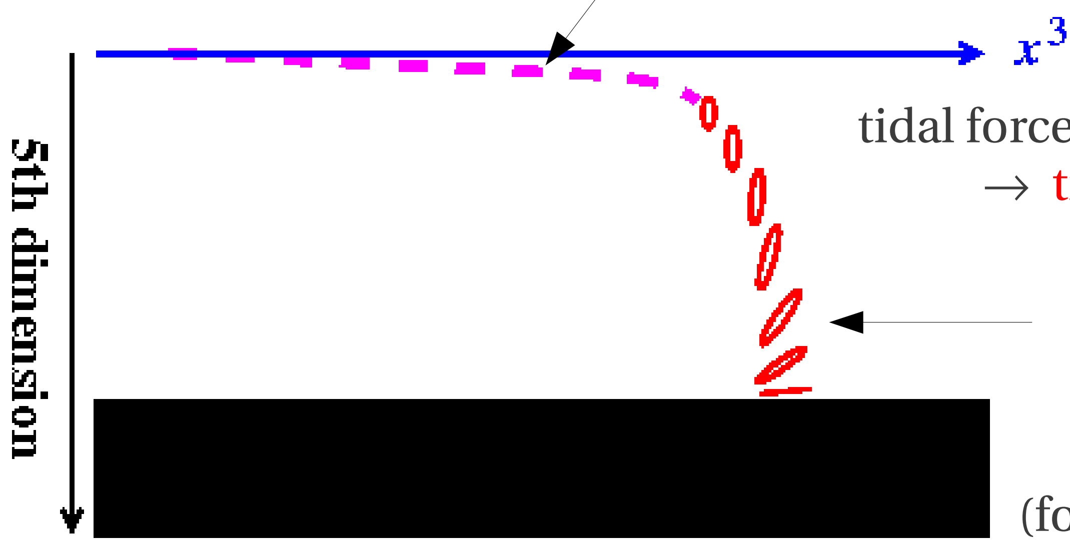

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Q: Is it possible to quantitatively calculate the late-time probability distribution of classical string configurations?

A: Yes, for a certain range of parameters.

Uses Penrose limit and quantization of strings in pp-wave backgrounds

AdS/CFT dictionary

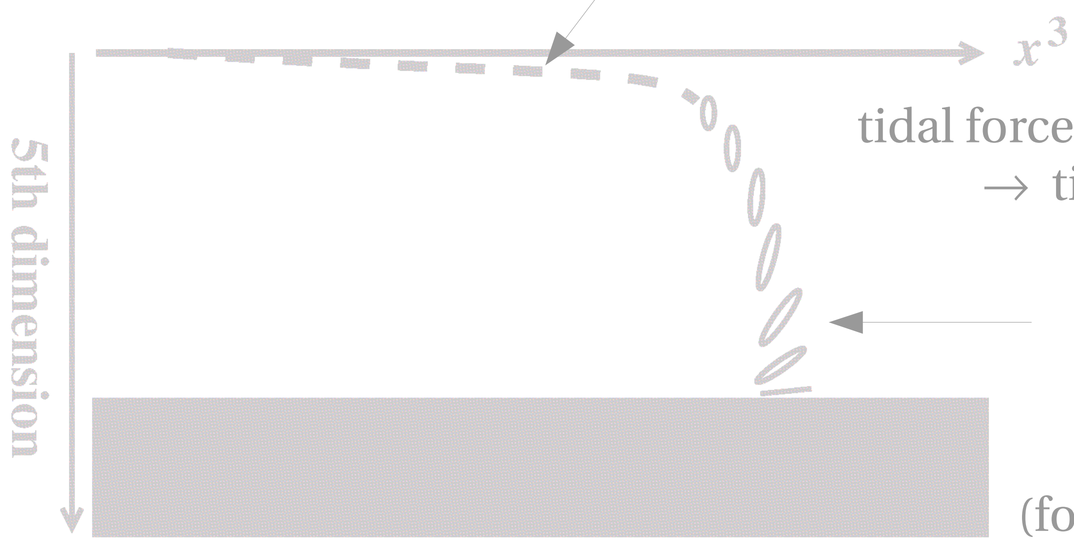
But really

● =  w/ internal degrees of freedom in ground state

proper size ~ (string tension)$^{-1/2}$ ~ $(\alpha')^{1/2}$

$\alpha' \propto \frac{1}{\sqrt{\lambda}}$

graviton is a quantum closed string



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$N_c \to \infty$

Q: Is it possible to quantitatively calculate the late-time probability distribution of classical string configurations?

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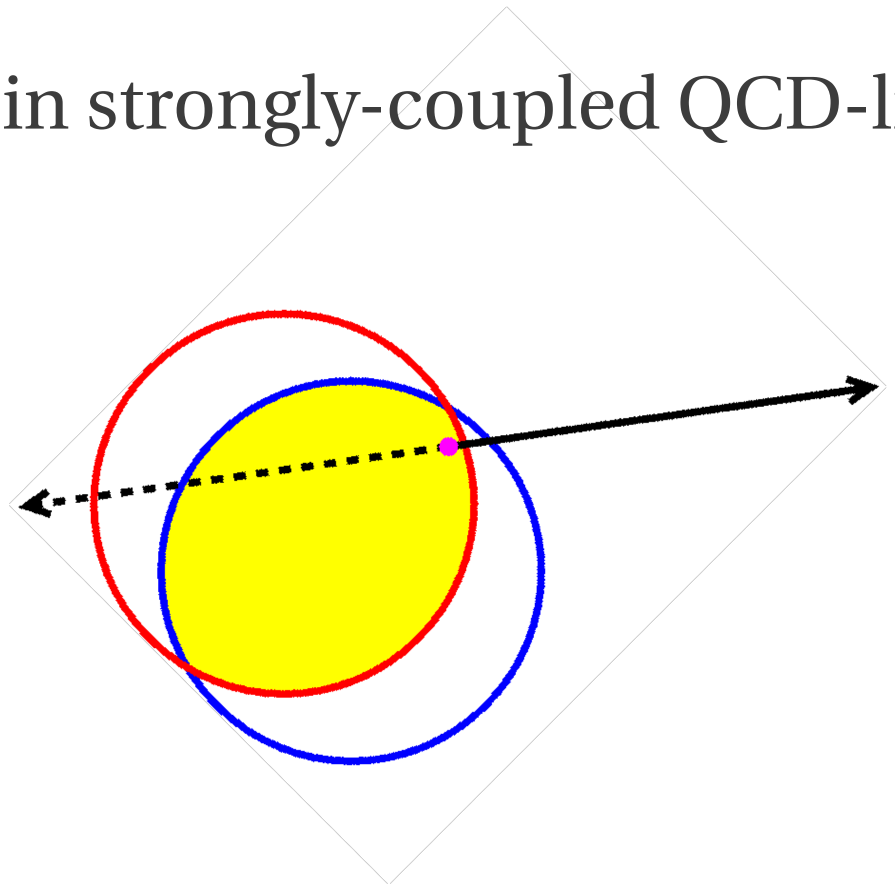
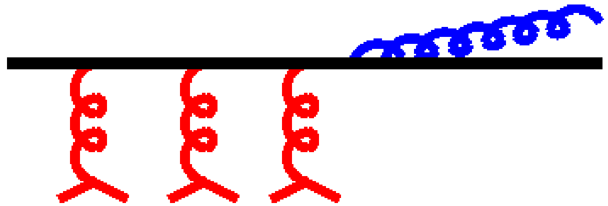
What's it good for?

Applied Holography: Problem arises in the theory of

Jet quenching in strongly-coupled QCD-like plasmas





$$Q_\perp \sim (\hat{q}E)^{1/4}$$

typical transverse momentum transfer during formation time

How stopping length scales with energy (massless case)

weak coupling: $\alpha_s \sim \alpha_s$ small — $\ell_{stop} \propto E^{1/2}$ (up to logs)

[this scaling a corollary of BDMPS and Z (1996)]

mixed coupling: α_s BIG, α_s small } — $\ell_{stop} \propto E^{1/2}$ (believed); $\ell_{stop} \sim \alpha_s^{-1}(E/\hat{q})^{1/2}$

[e.g. Liu, Rajagopal, Wiedeman (2006)]

all strong coupling: $\alpha_s = \alpha_s$ BIG — $\ell_{stop} \propto E^{1/3}$
(Large-N_c $\mathcal{N}$=4 SYM, etc.)

[Gubser, Gollota, Pufu, Rocha; Hatta, Iancu, Mueller; Chesler, Jensen, Karch, Yaffe (2008)]

Interesting: Exponent in $\ell_{stop} \propto E^{\nu}$ can depend on α_s.

Defining Stopping Distance

BIG α_s=α_s: Large-N_c $\mathcal{N}$=4 SYM, etc. with $N_c\alpha_s \to \infty$

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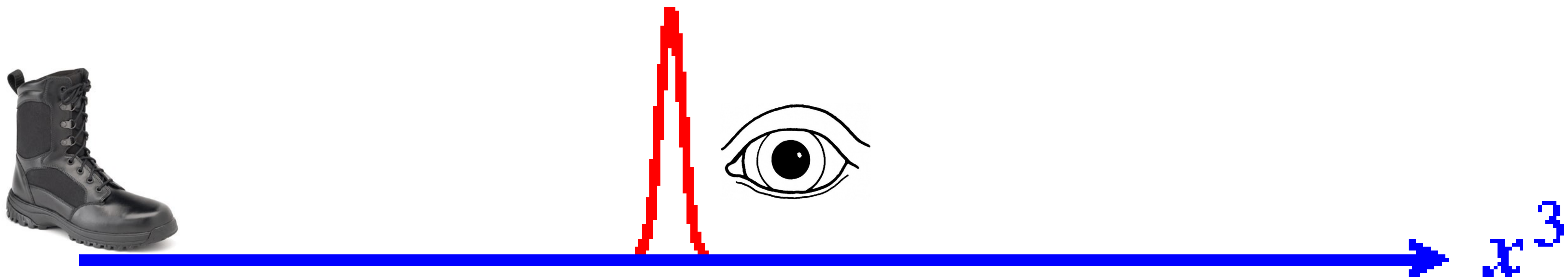

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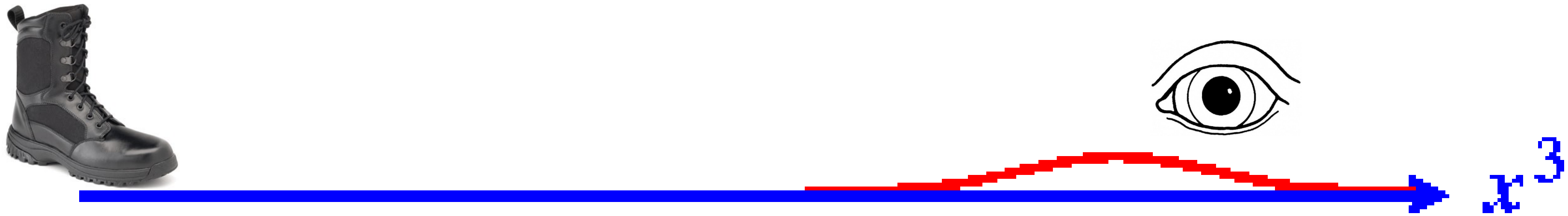

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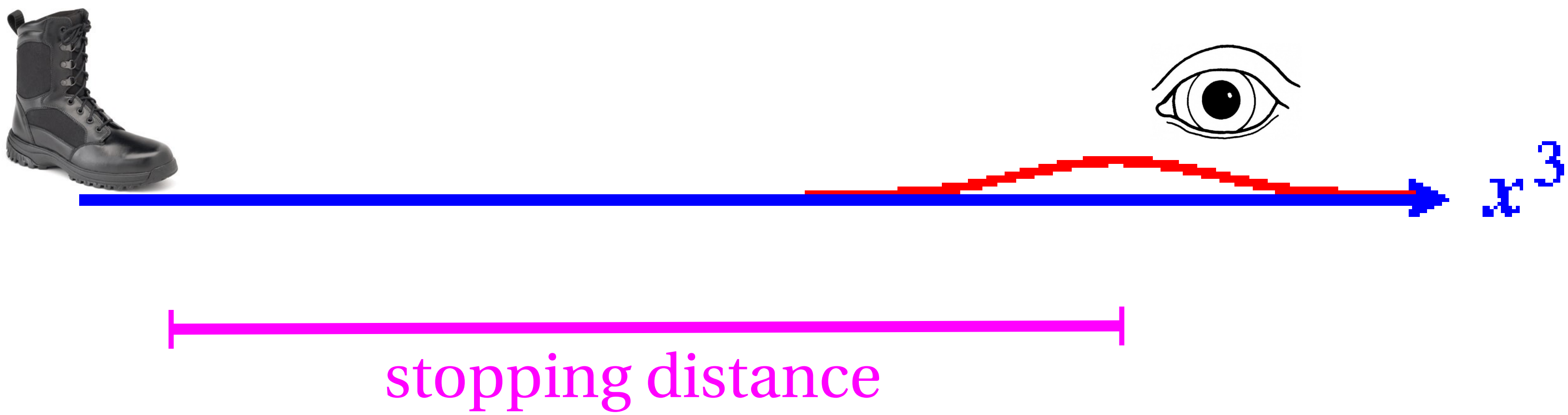


AdS/CFT Results

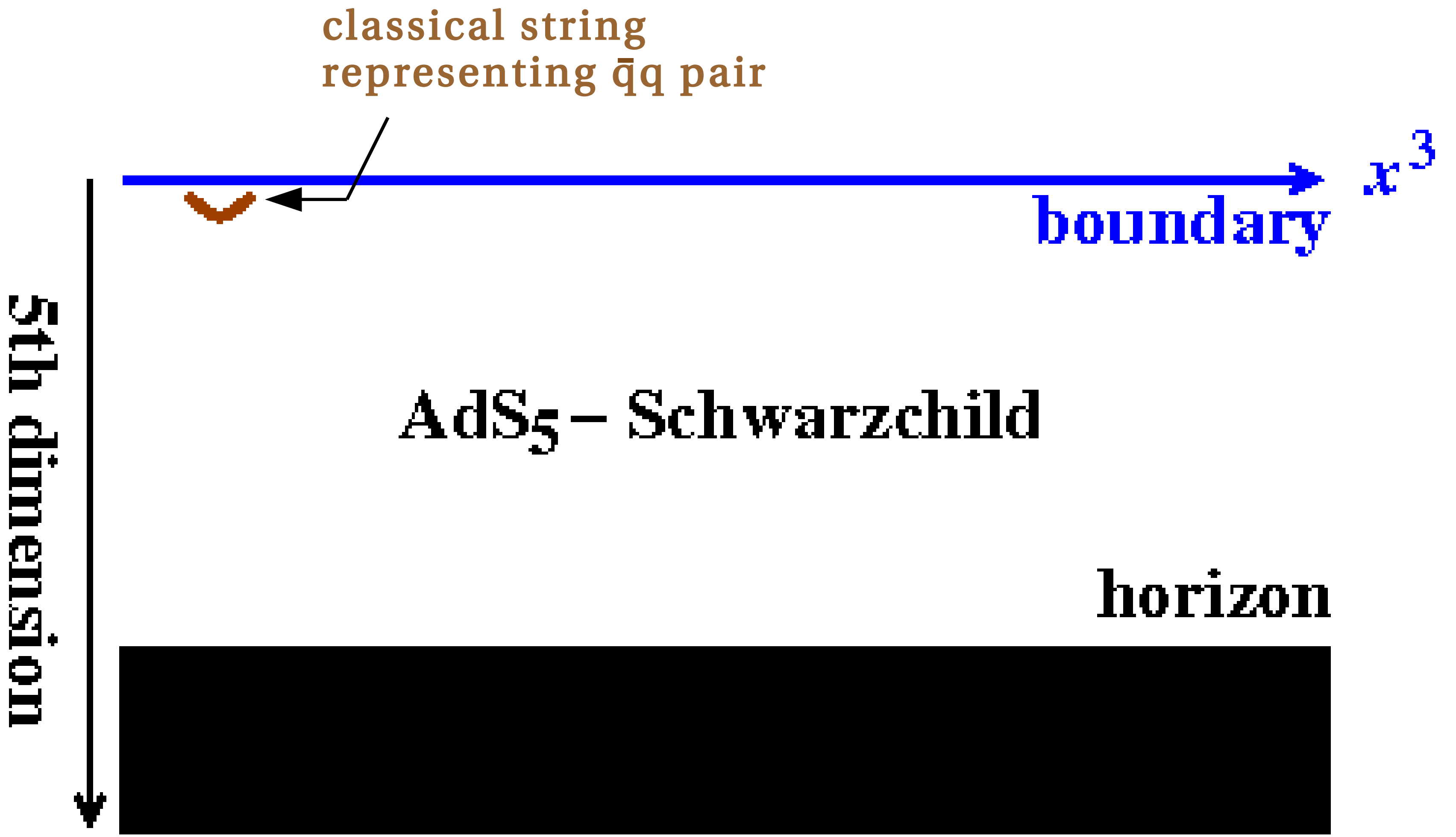
A. Results for $\lambda = \infty$ and $N_c = \infty$

B. Results for $1 << \lambda < \infty$ and $N_c = \infty$

Early AdS Calculations

Example: Classical string calculation of

Chesler, Jensen, Karch, Yaffe (2008)



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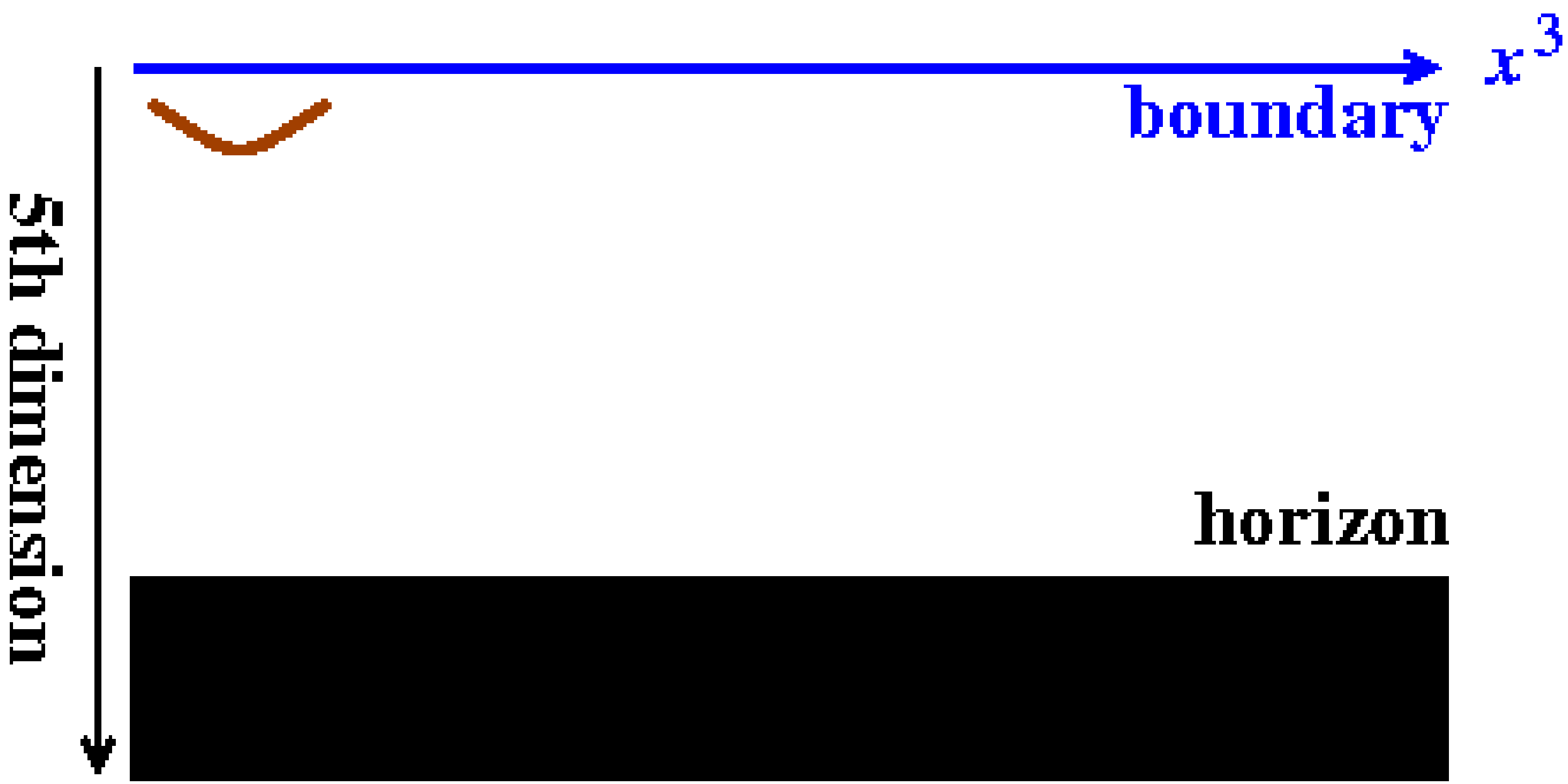

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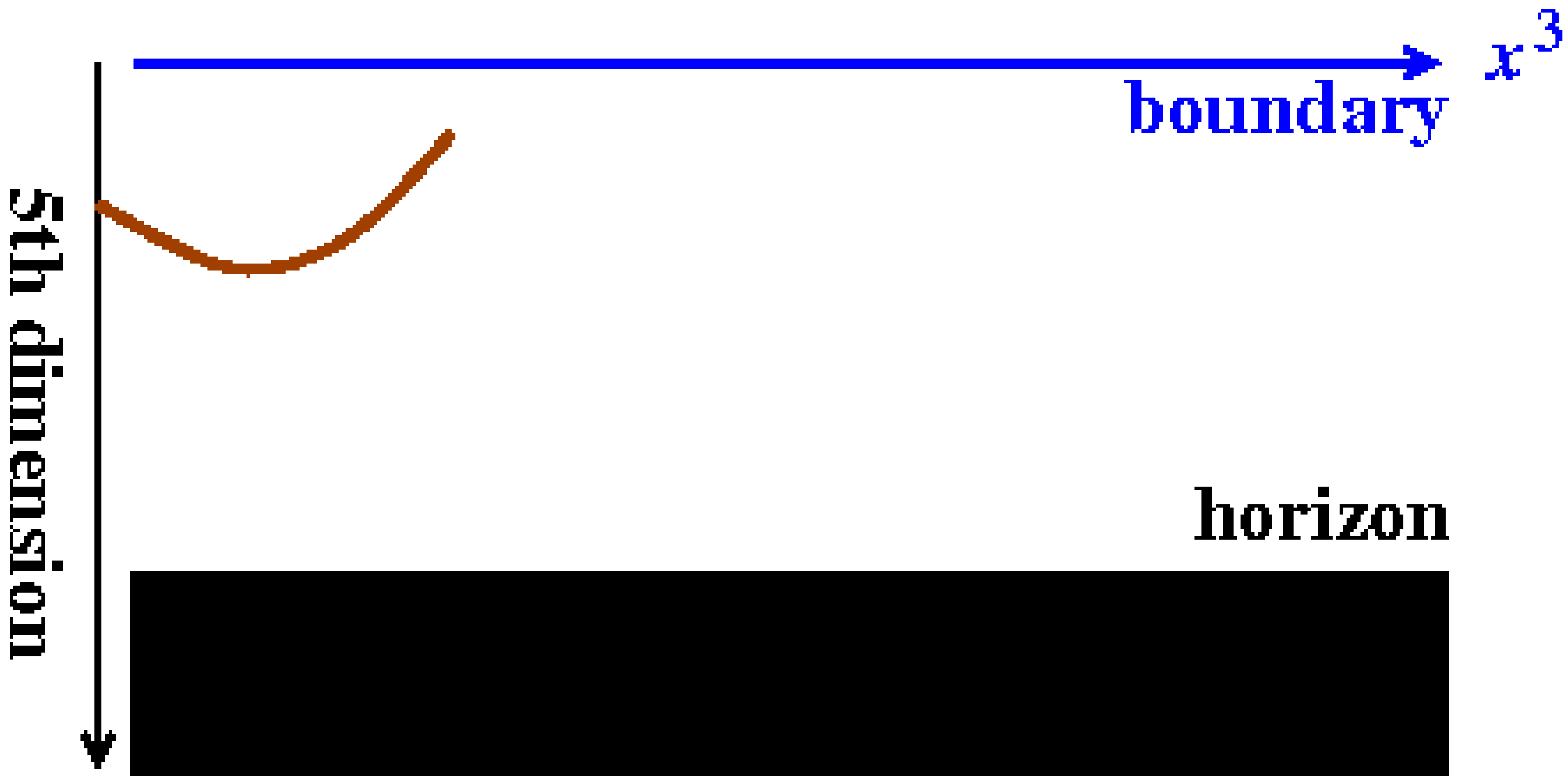

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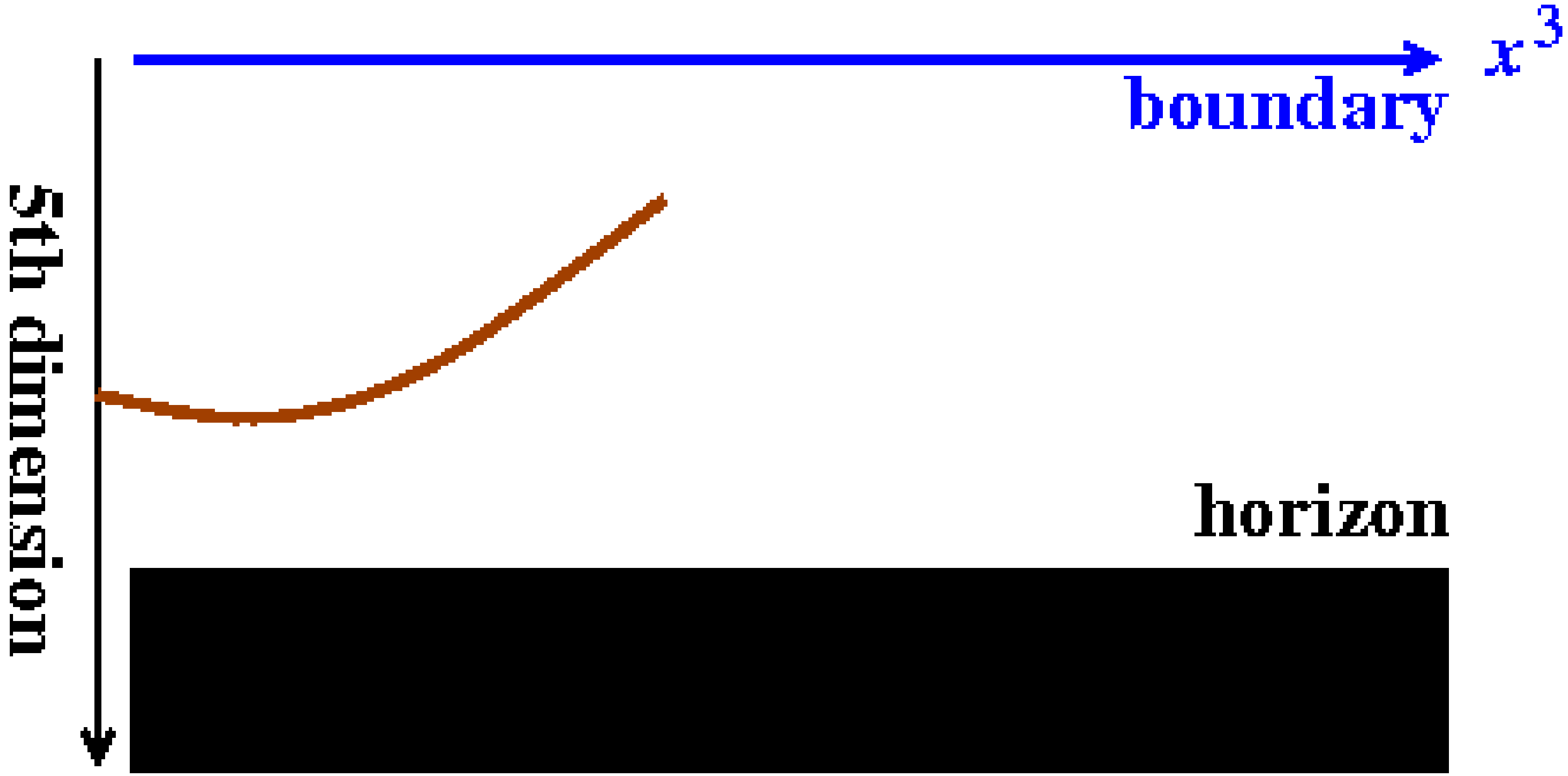

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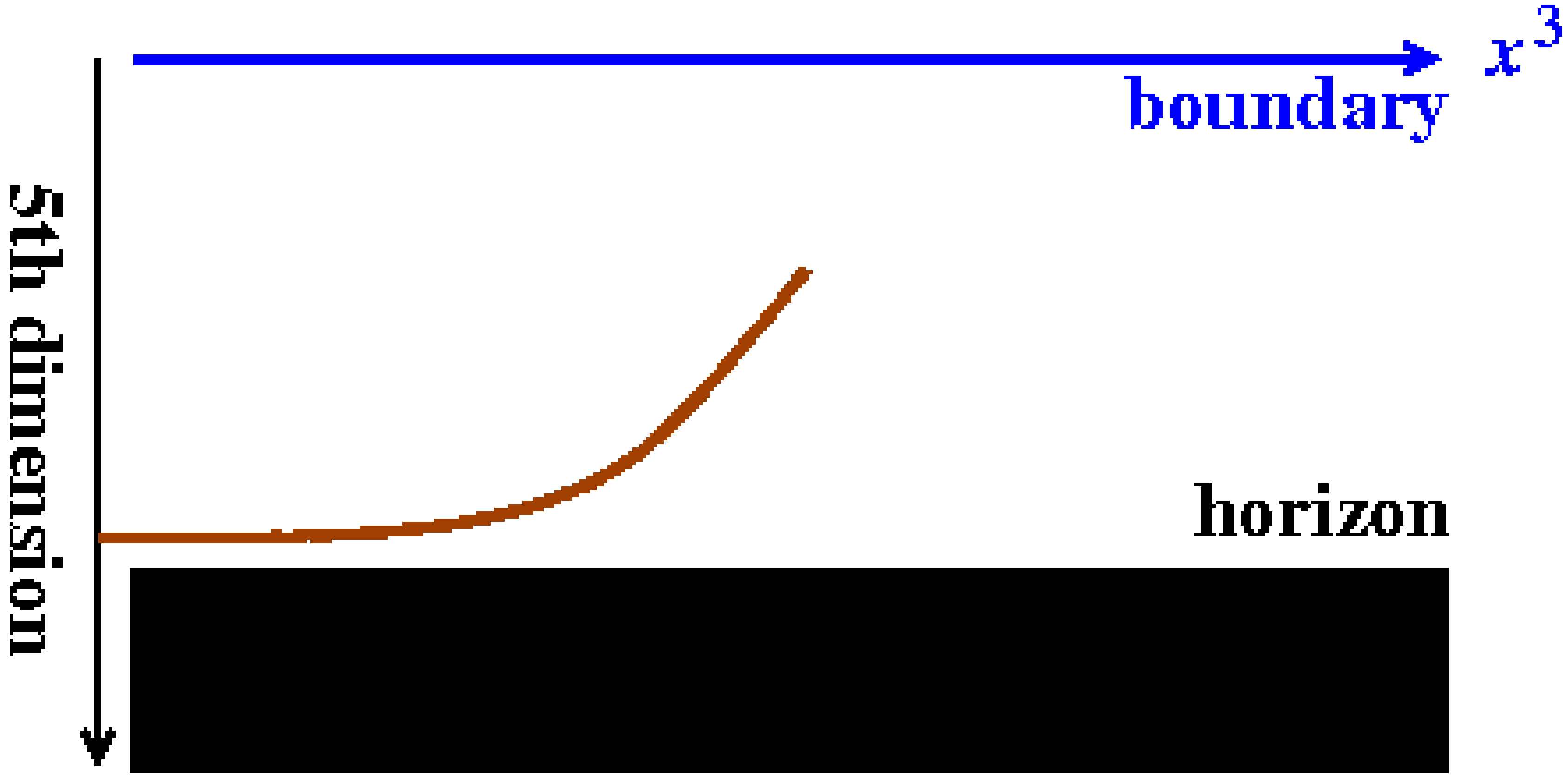

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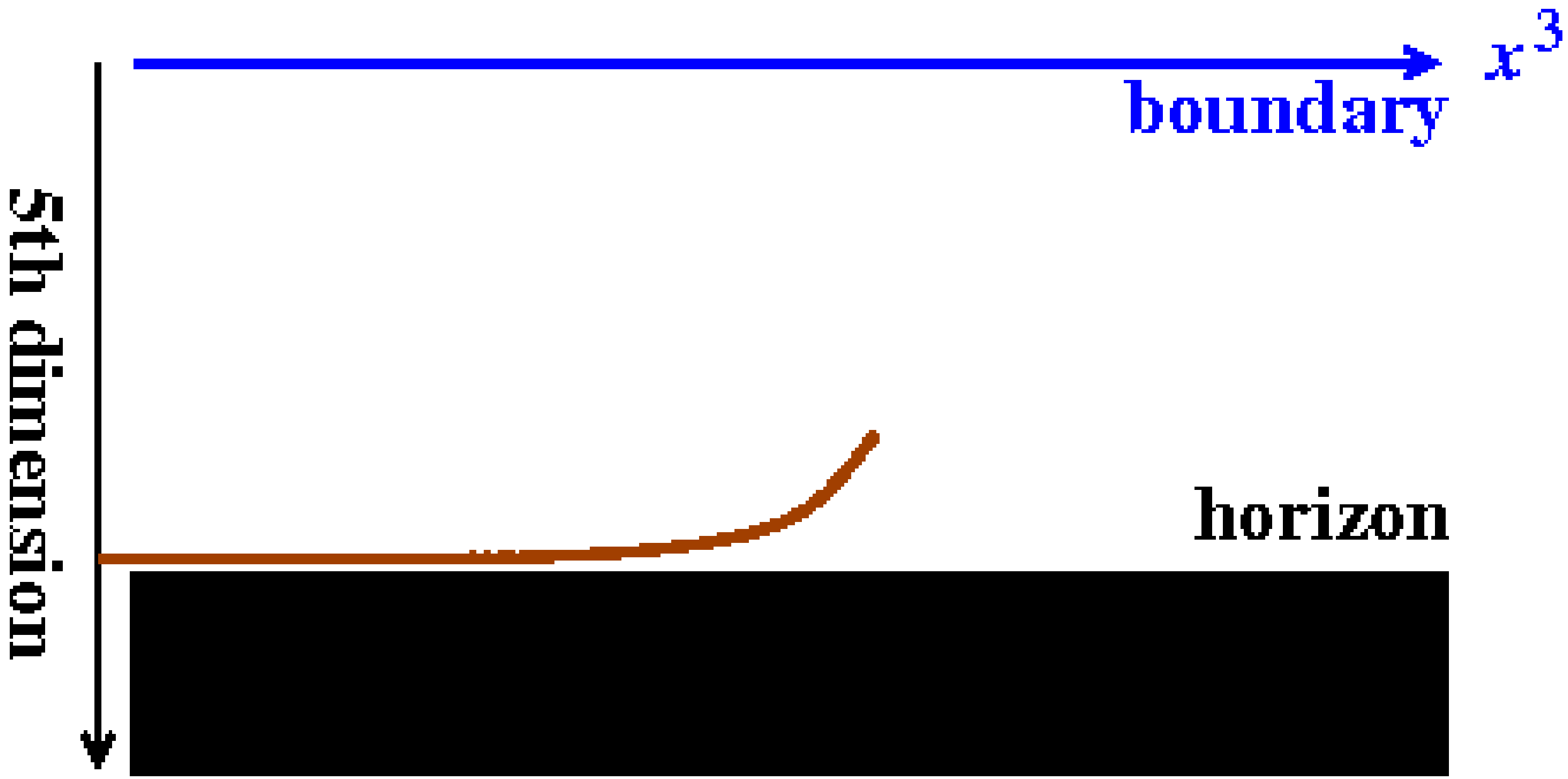

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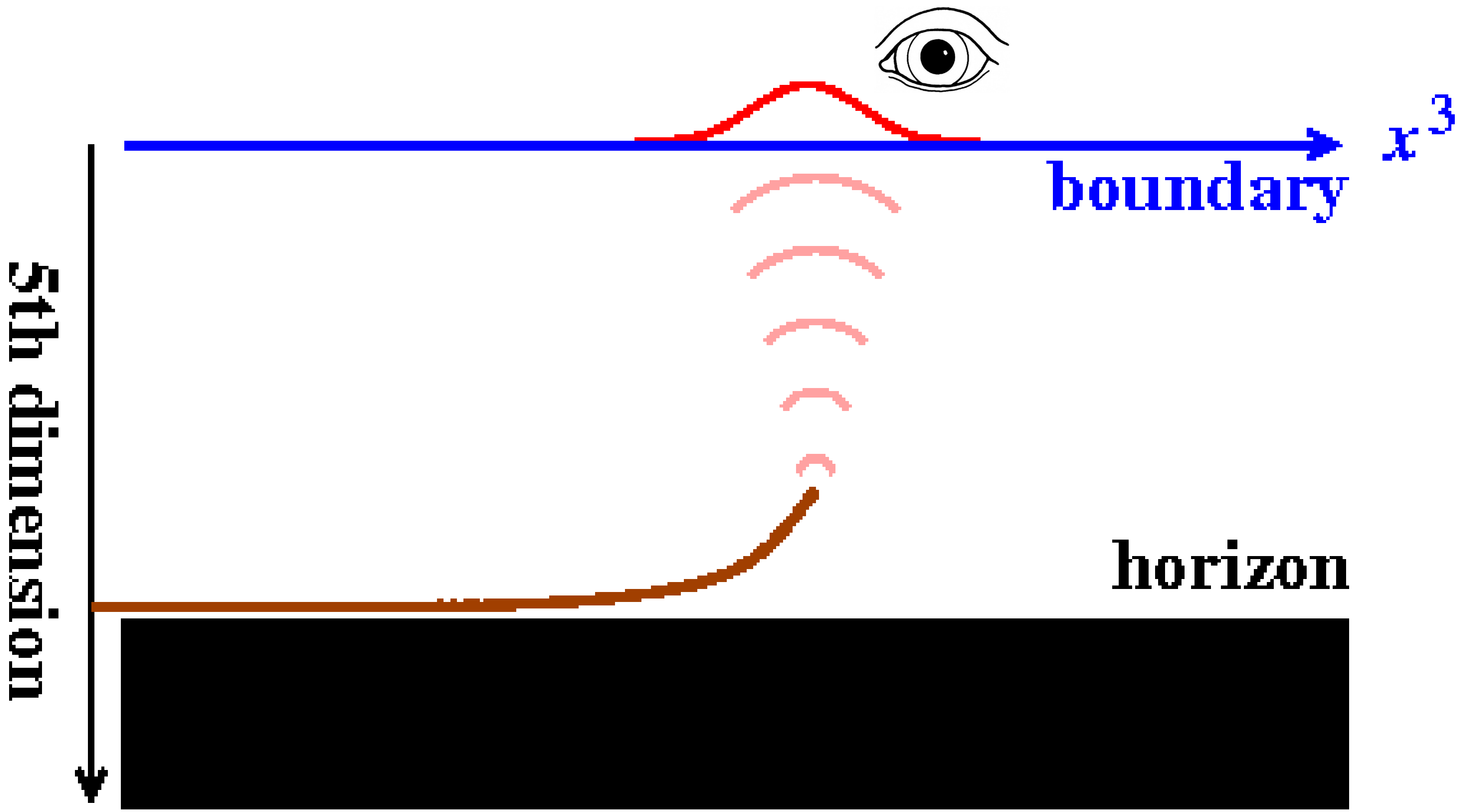
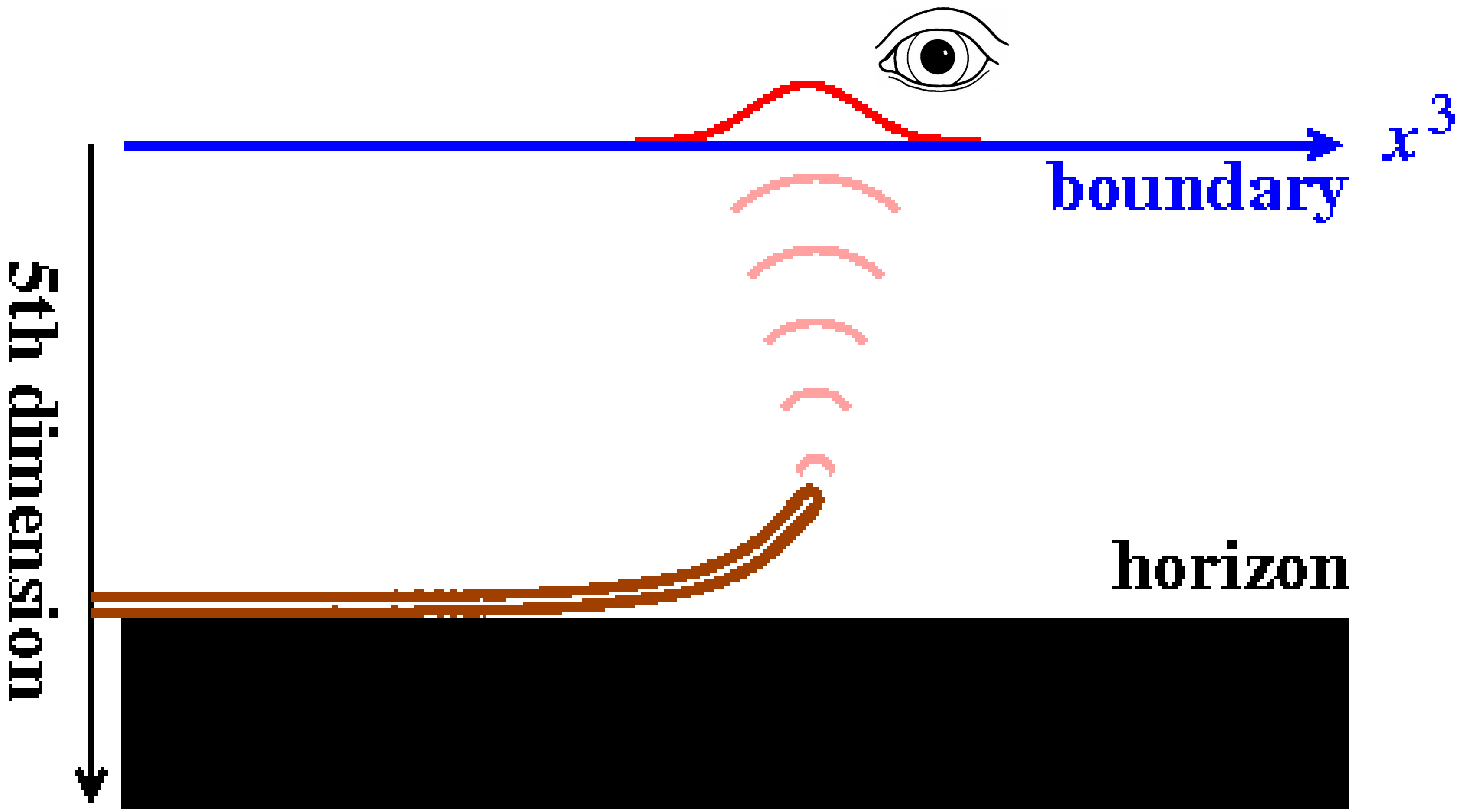


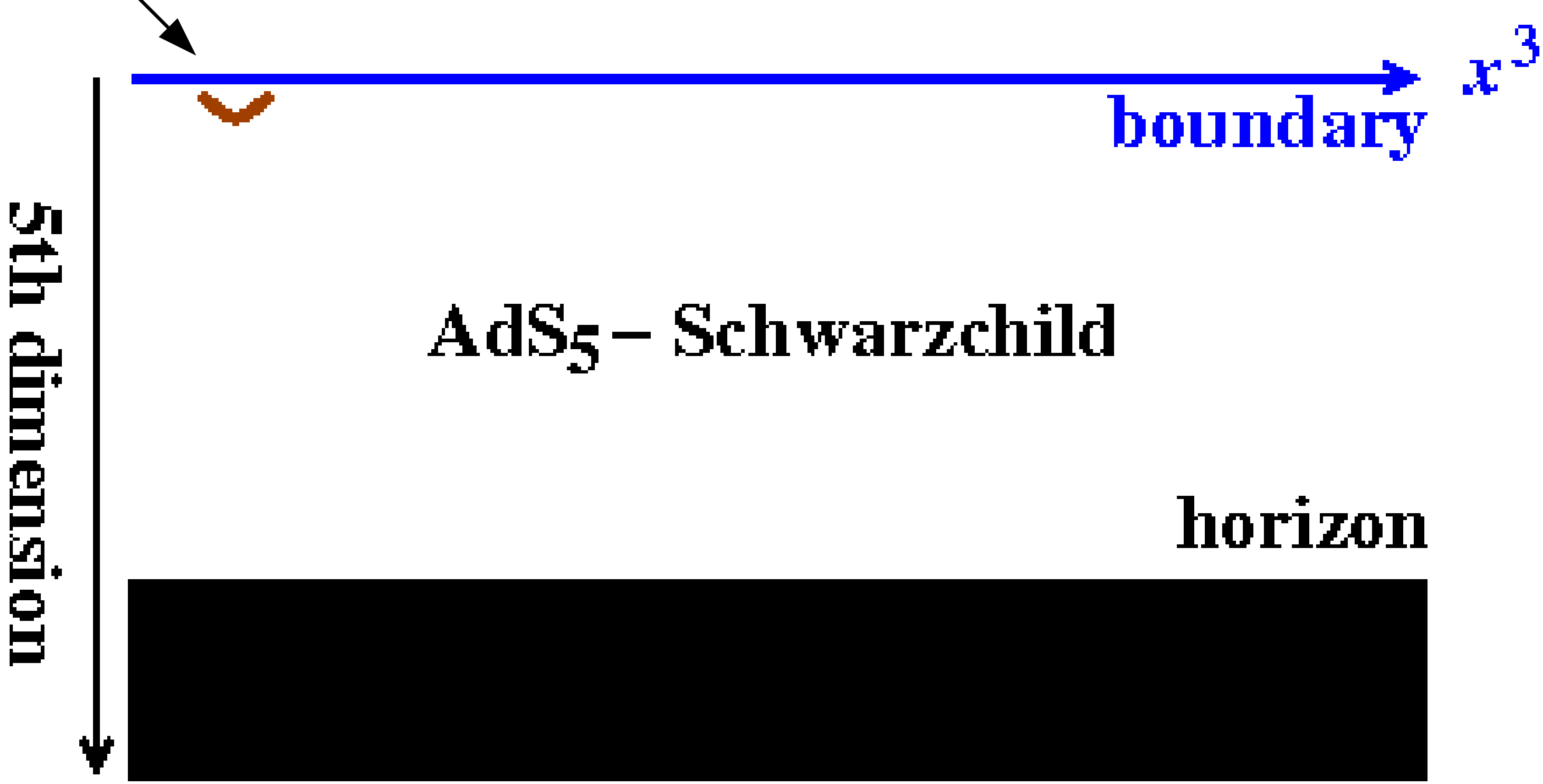
Early AdS Calculations

Gubser, Gulotta, Pufu, Rocha (2008) had a roughly similar picture but with a folded string, representing a gluon jet.



Something slightly dissatisfying: What is the  ?

The initial configuration has been expressed in the gravity dual.
How precisely do I set up the problem in the 3+1 dim. field theory?



Choice of  in the QFT

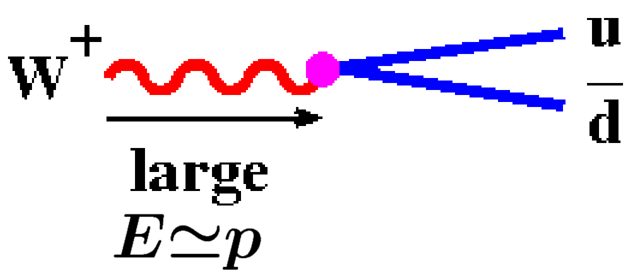
Gedanken experiments for creating localized, very high-momentum excitations in the plasma.

Synchrotron method: Drag a heavy test quark around in a circle to make a beam of gluon synchrotron radiation.

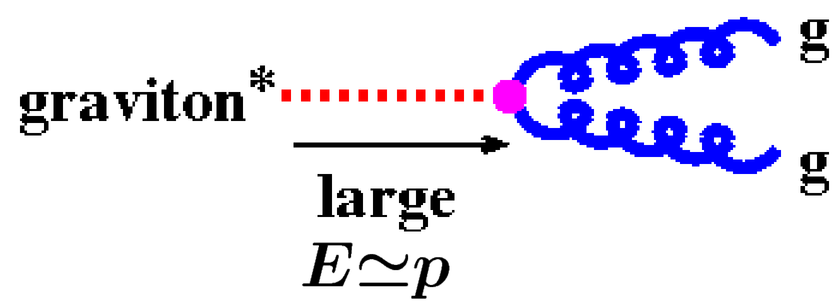
[used by Chesler, Ho, Rajagopal (2011)]

Our method: Analogous to considering the hadronic decay of some very high-momentum, unstable particle in a QCD plasma.

e.g.



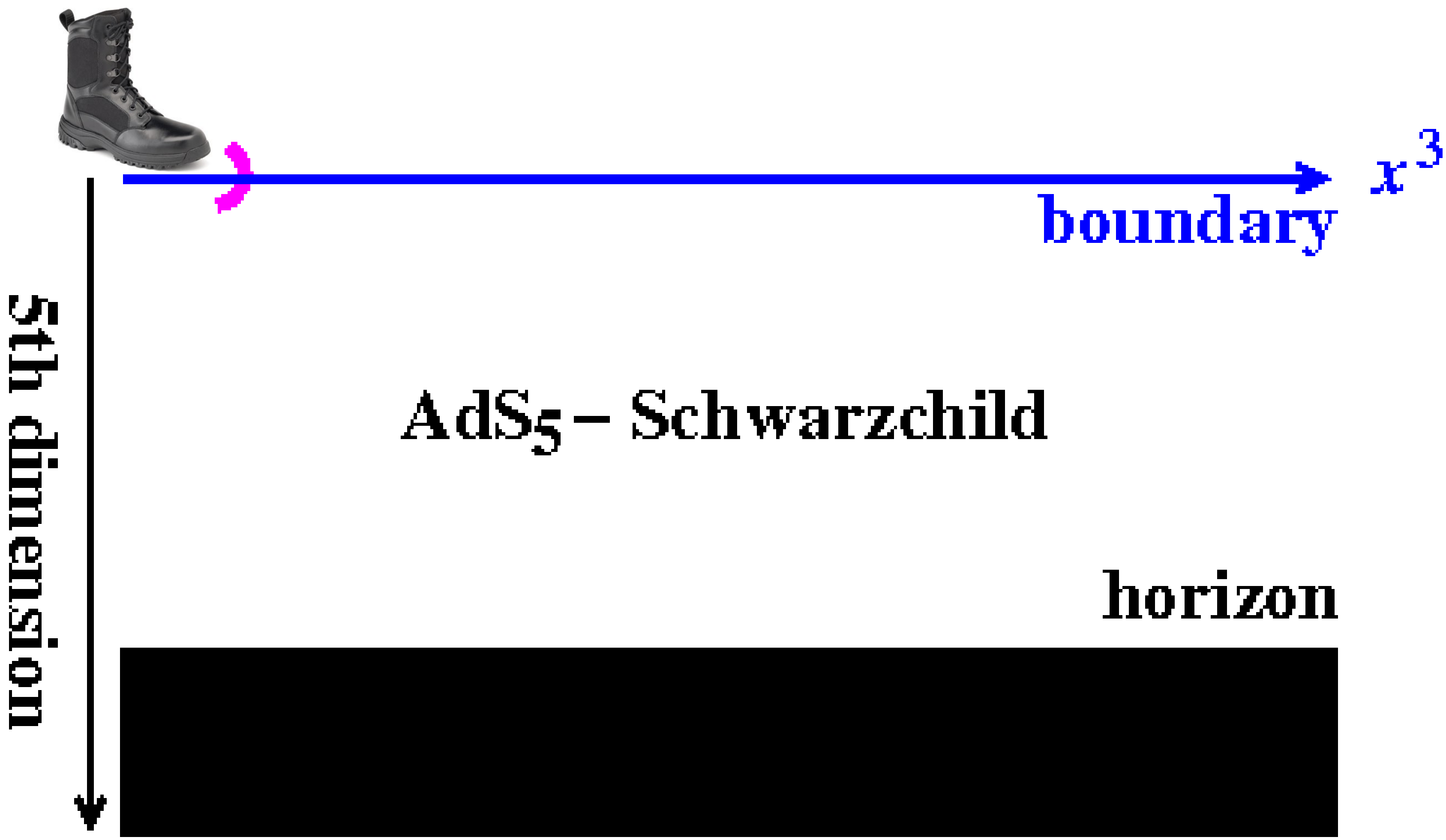
or



[Arnold & Vaman (2010)]

Stopping Distance using AdS/CFT

BIG $\alpha_s = \alpha_s$:   Large-N_c $\mathcal{N}$=4 SYM, etc. with $N_c \alpha_s \to \infty$



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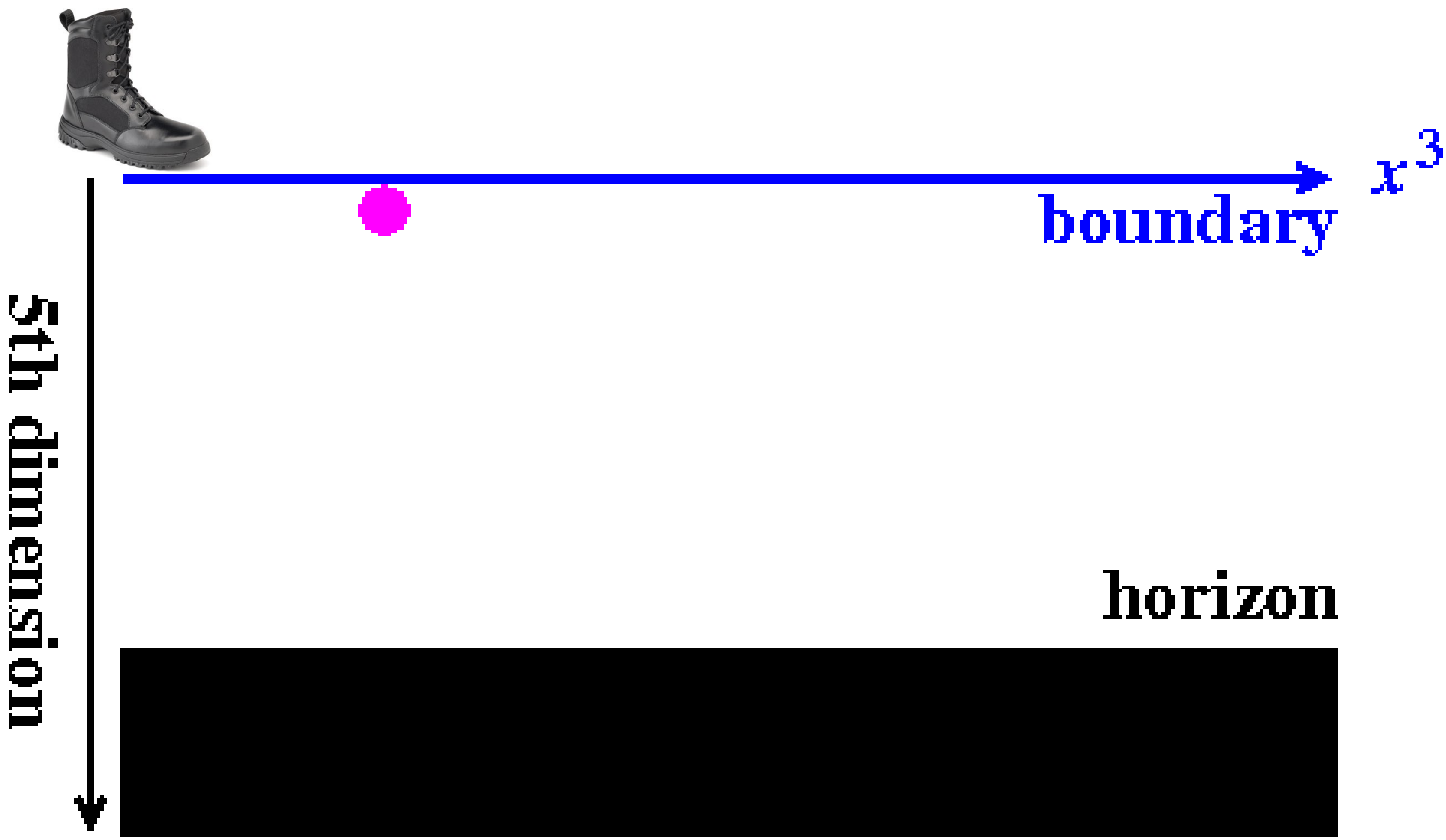

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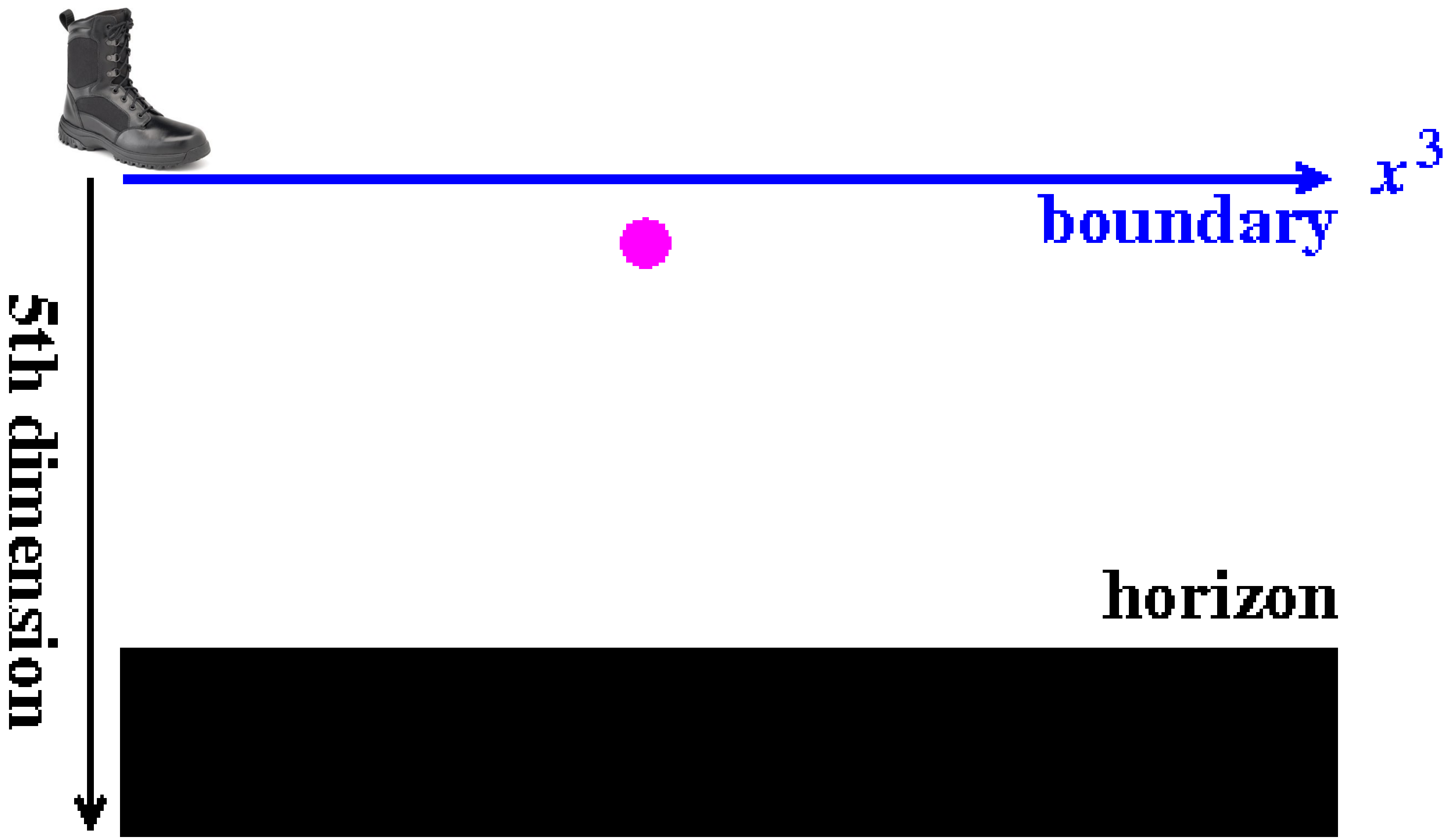

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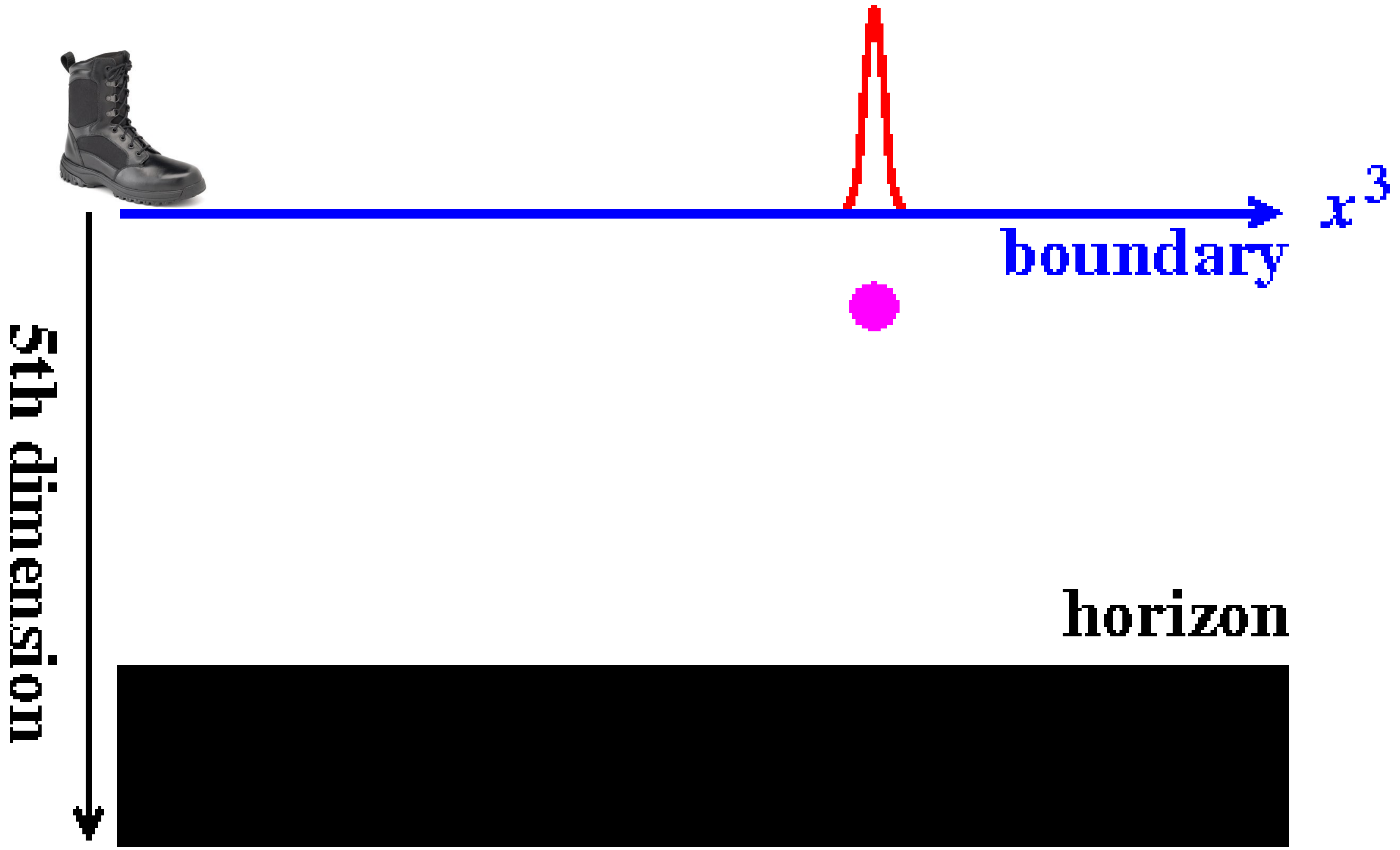

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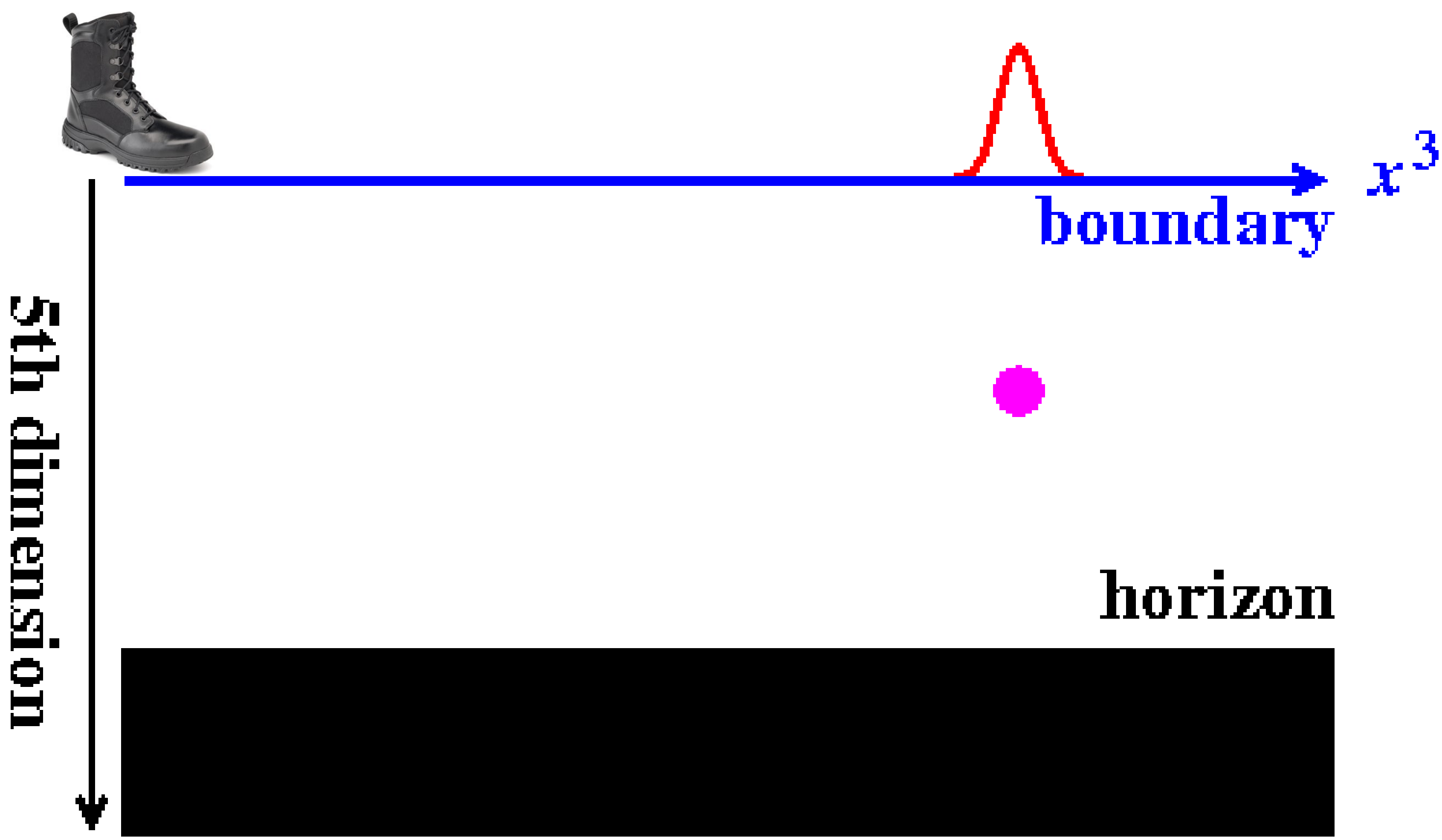

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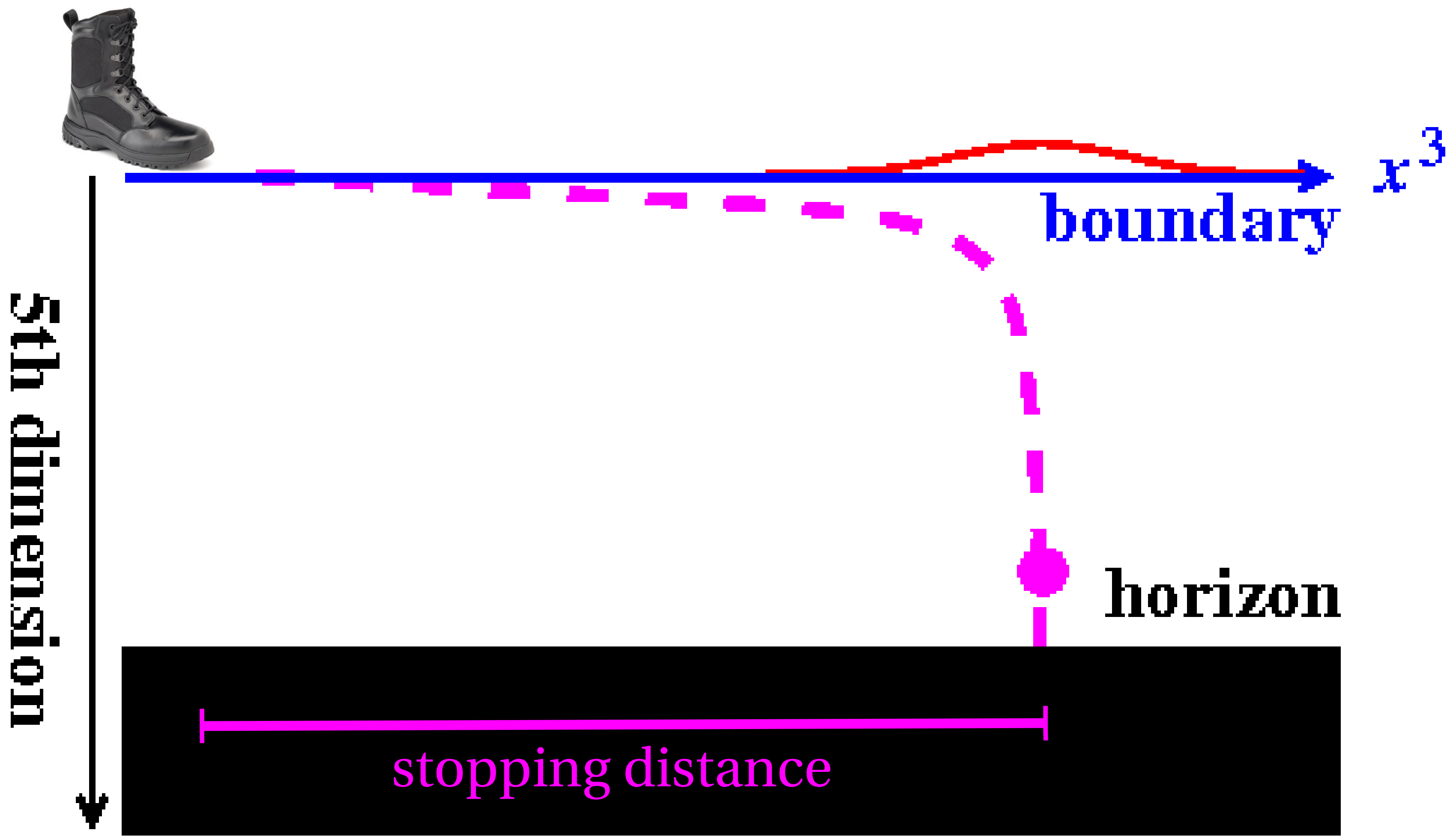
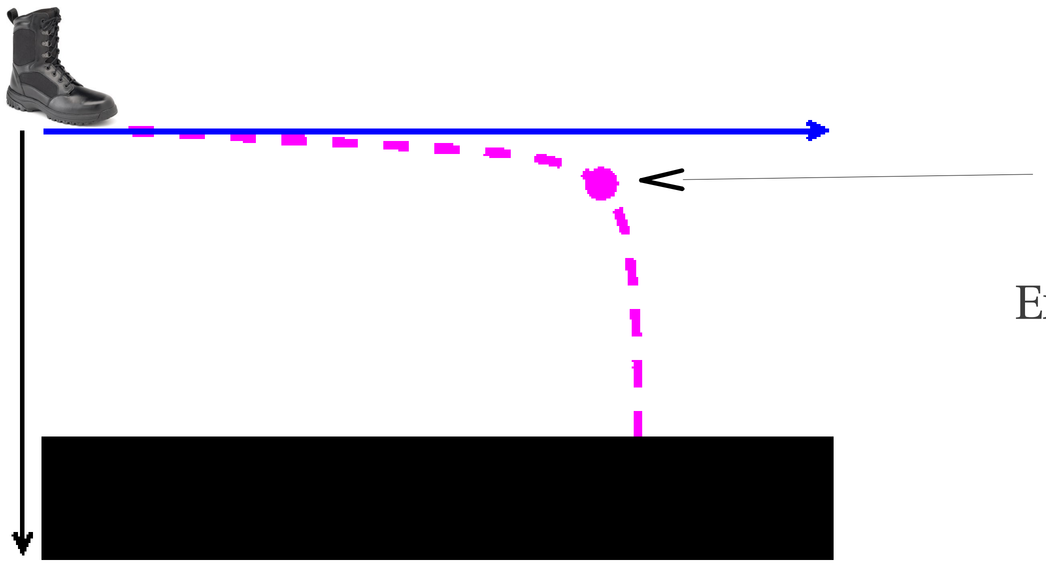


$\lambda = \infty$ result

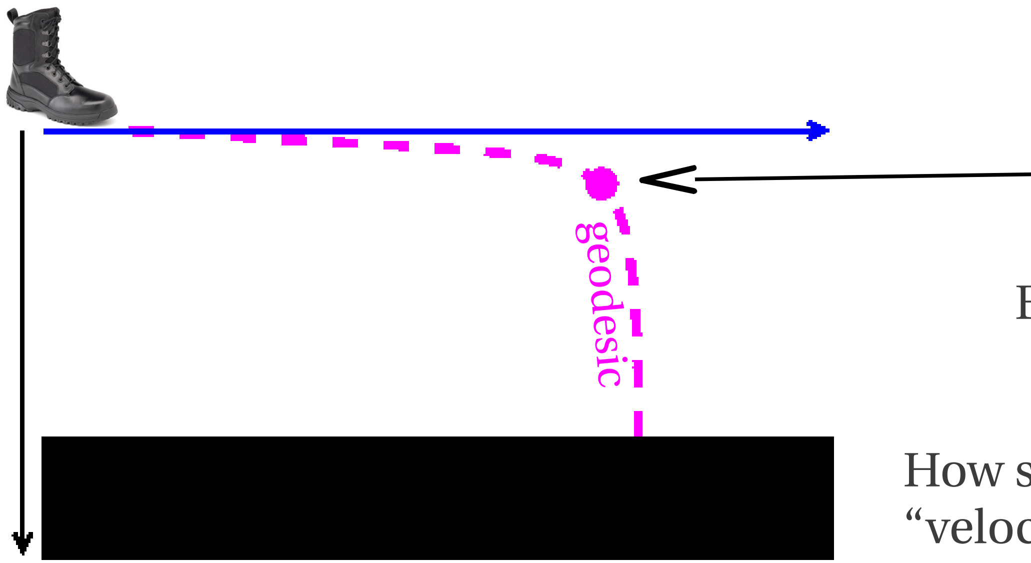
$$\boldsymbol{\ell_{\mathrm{stop}} \quad \lesssim \quad \ell_{\mathrm{max}} \propto E^{1/3}}$$

A simplified picture for $\ell_{\mathbf{stop}} \ll \ell_{\mathbf{max}}$



Excitation moves like a 5-dim. particle!

A simplified picture for $\ell_{\rm stop} \ll \ell_{\rm max}$



small size

Excitation moves like a 5-dim. particle!

How soon it falls will depend on the initial "velocity" in the $5^{\rm th}$ dimension.

Q: What determines $\ell_{\rm stop}$?

A: the 4-virtuality $q^2 \equiv q_\mu \eta^{\mu\nu} q_\nu$ of the source 

Why?

Consider massless 5-dim. particle near the boundary:

$$0 = q_\mu q^\mu + q_5 q^5$$

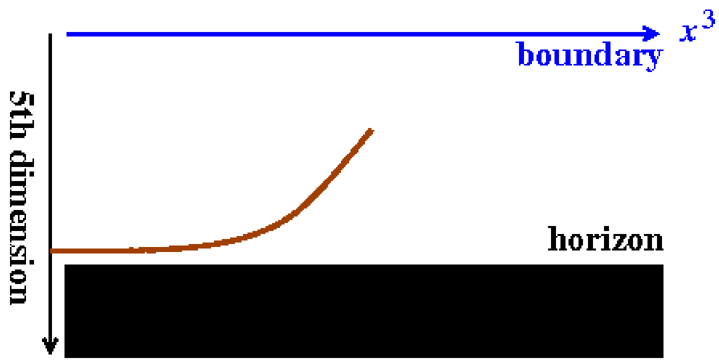
Bigger $-q_\mu q^\mu \rightarrow$ bigger $q^5 \rightarrow$ falls sooner !

The result:

$$\ell_{\rm stop} \simeq \frac{\Gamma^2(\frac{1}{4})}{\sqrt{4\pi}} \left(\frac{E^2}{-q^2} \right)^{1/4}$$

A difference between different treatments

“Jets” represented as classical strings in dual theory

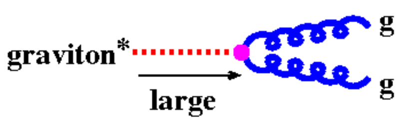


$$\ell_{\rm max} \sim T^{-4/3} (E/\sqrt{\lambda})^{1/3}$$

Gubser, Gulotta, Pufu, Rocha (2008)
Chesler, Jensen, Karch, Yaffe (2008)

“Jets” that are not

e.g.



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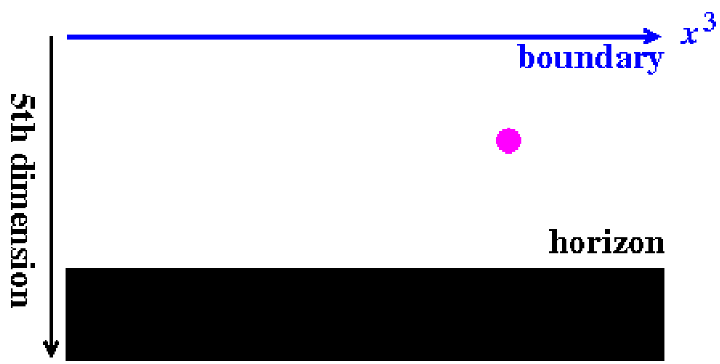


$$\ell_{\rm max} \sim T^{-4/3} E^{1/3}$$

Hatta, Iancu, Mueller (2008)
Arnold & Vaman (2010)
Chesler, Ho, Rajagopal (2011)

AdS/CFT Results

A. Results for $\lambda = \infty$ and $N_c = \infty$

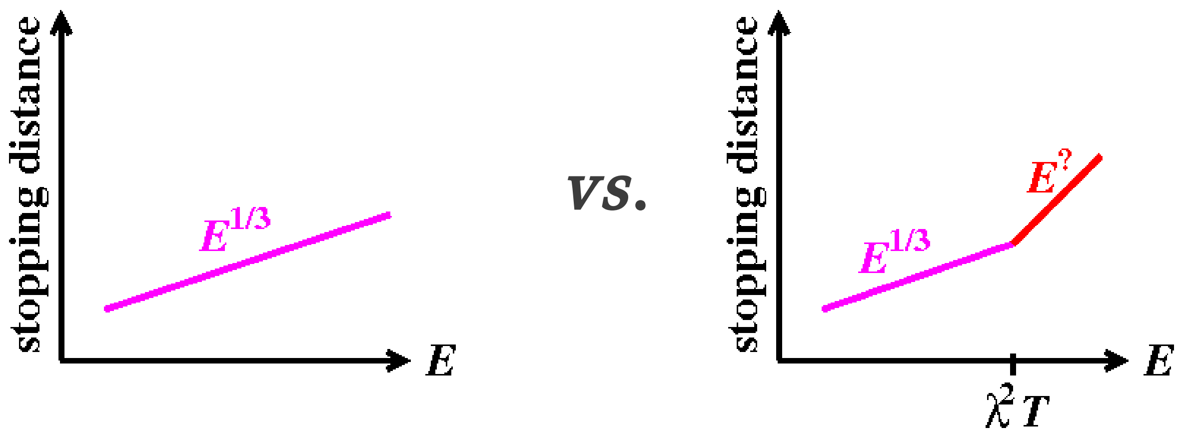
B. Results for $1 << \lambda < \infty$ and $N_c = \infty$

Previous "$\alpha_s = \alpha_s$ BIG" result $\ell_{stop} \propto E^{1/3}$ has only been derived for $\lambda = N_c\alpha = \infty$.

What could we learn by also studying λ BIG but $< \infty$?

Answer: Is the high-energy behavior <u>really</u> $E^{1/3}$?

e.g.



(*Note:* will assume $N_c = \infty$ throughout.)

Our first look at this question

Arnold, Vaman, Szepietwoski, 1203.6658

1. Assume $1/\lambda^n$ corrections are small.
2. Estimate their size.
3. See if anything goes wrong as $E \to \infty$!

Higher curvature corrections to gauge-gravity duality

AdS/CFT correspondence:

$\mathcal{N}$=4 SYM ⟺ string theory in $AdS_5 \times S^5$ background

Strong-coupling limit:

$\lambda \to \infty$ ⟺ "low energy" string theory
= supergravity in in $AdS_5 \times S^5$ background
(gravitons + other massless string modes)

$$\mathcal{L}_{\mathbf{grav}} \sim R$$

Higher curvature corrections to gauge-gravity duality

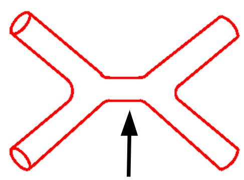
AdS/CFT correspondence:

$\mathcal{N}$=4 SYM $\Longleftrightarrow$ string theory in $AdS_5 \times S^5$ background

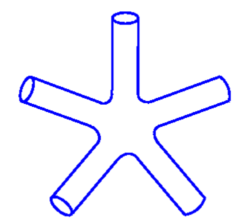
Strong-coupling limit:

$\lambda \to \infty$ $\Longleftrightarrow$ "low energy" string theory
= supergravity in in $AdS_5 \times S^5$ background
(gravitons + other massless string modes)

$$\mathcal{L}_{\mathbf{grav}} \sim R \quad + \quad \alpha'^3 R^4 + \alpha'^5 D^4 R^4 + \alpha'^6 D^6 R^4 + \cdots \quad + \quad \alpha'^5 D^2 R^5 + \alpha'^6 D^4 R^5 + \cdots \quad + \quad \cdots$$



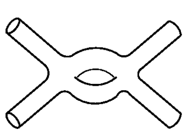
integrate out massive
intermediate string states



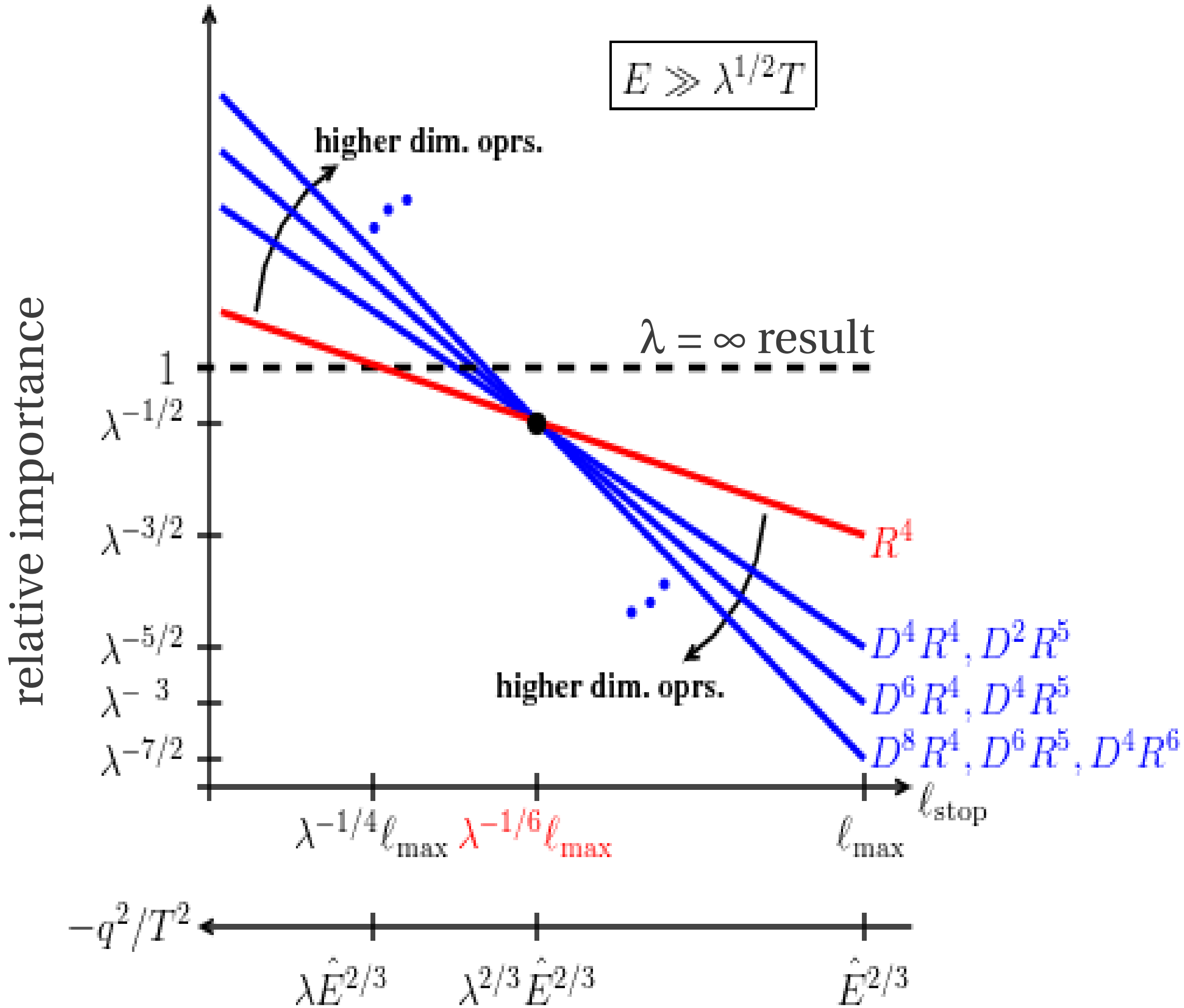
ditto

where

$\dfrac{1}{\sqrt{\lambda}}$ $\Longleftrightarrow$ proportional to $\alpha' \sim \dfrac{1}{\text{string tension}}$

Note: Loops  are suppressed by $g_{\mathbf{string}} \propto \dfrac{1}{N_c}$.

Importance to Jet Stopping



Reminder:

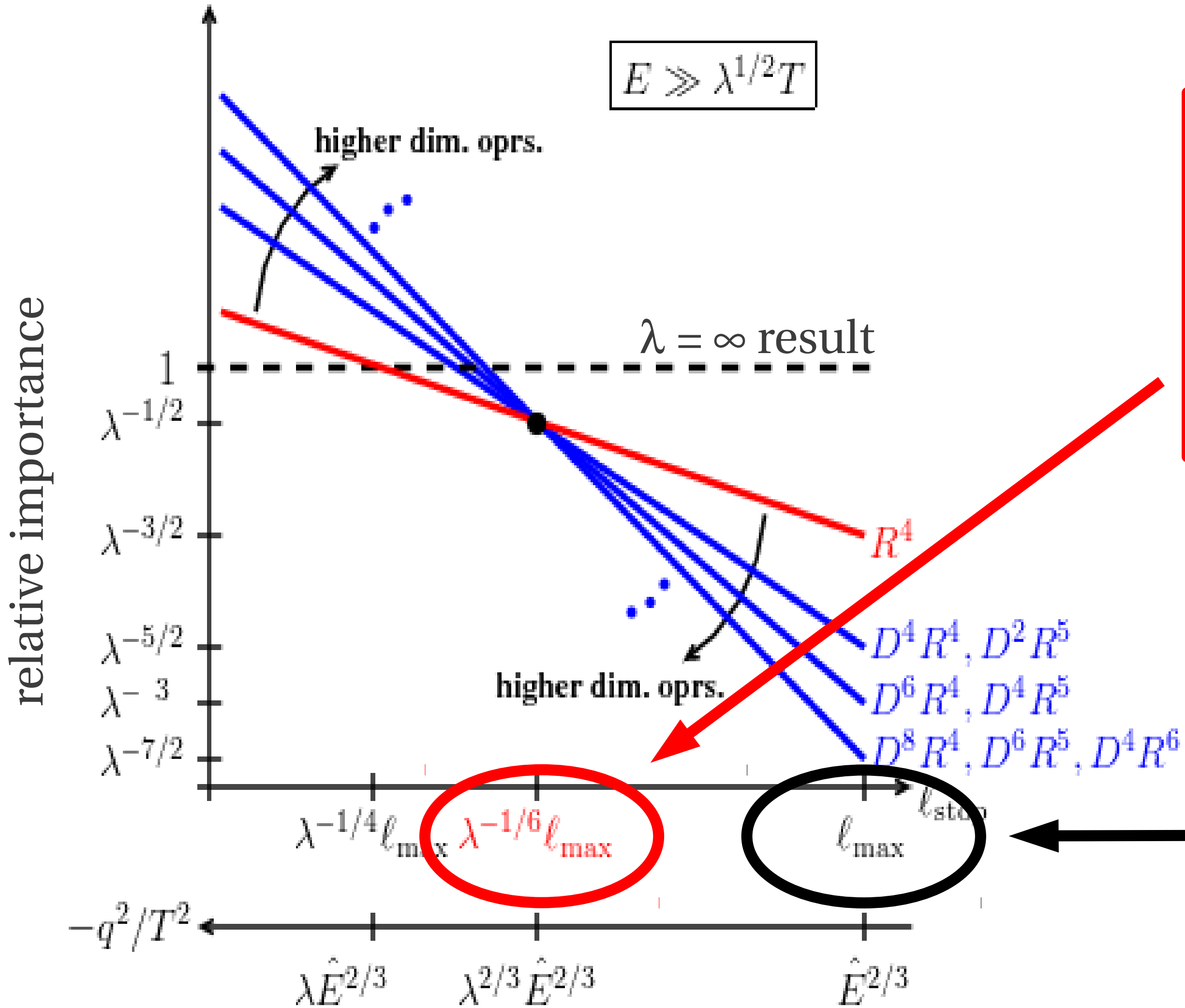
$\ell_{max} \propto E^{1/3}$

Moral: Expansion in $1/\lambda$ is well-behaved for $\lambda^{-1/6}\ell_{max} \ll \ell_{stop} \lesssim \ell_{max}$

Expansion breaks down for $\ell_{stop} \lesssim \lambda^{-1/6}\ell_{max}$

Note: Individual corrections all small ($\lambda^{-1/2}$) where expansion first breaks down.

Importance to Jet Stopping



This is

$$T^{-4/3}(E/\sqrt{\lambda})^{1/3}$$

the max stopping distance for classical strings!

Reminder:

$$\ell_{max} \sim T^{-4/3} E^{1/3}$$

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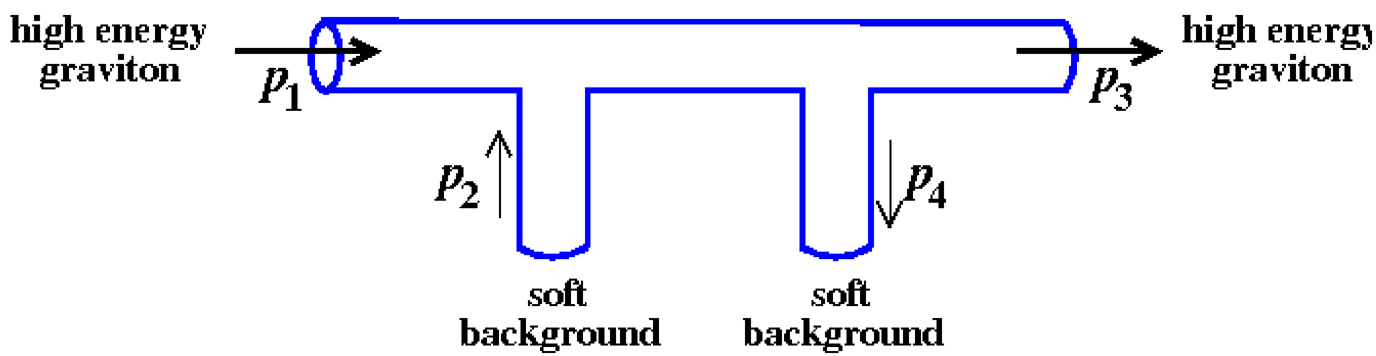
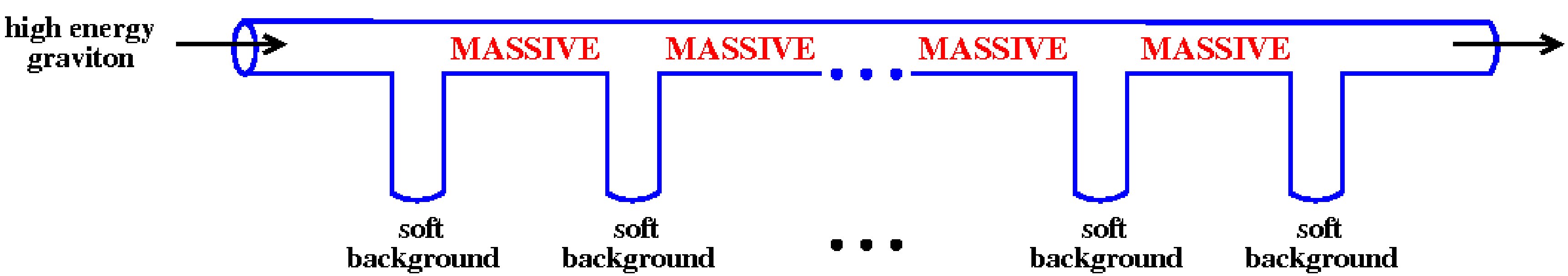
So what's gone wrong?



5-dimensional $\sqrt{s} \gtrsim M_{\text{stringy}}$

→ can excite real, massive, stringy internal states on-shell !

→ small p expansion breaks down, and also



is unsuppressed.

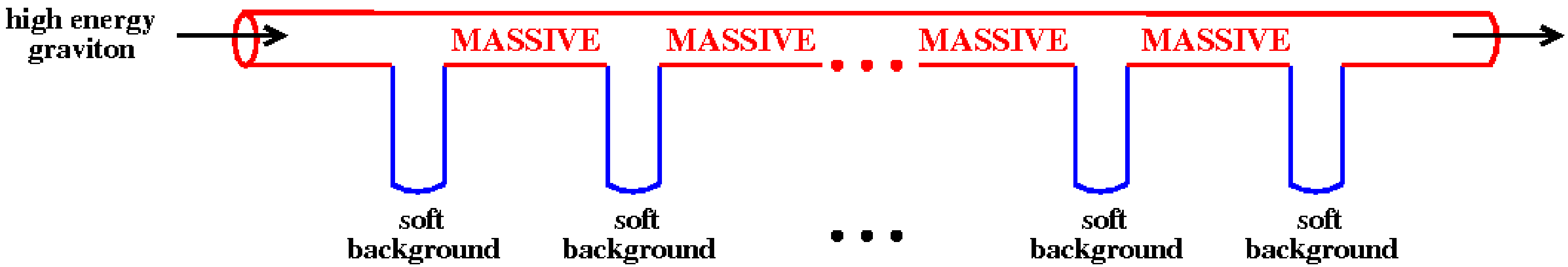
When might this possibly happen?

$\sqrt{s} \propto \sqrt{E}$ and $M_{\text{stringy}} \propto (\text{tension})^{1/2} \propto (\alpha')^{-1/2} \propto \lambda^{1/4} \quad \Rightarrow \quad E \gg \sqrt{\lambda}\, T$

Details about l_{stop} condition comes from whether stringy modes excited before or after stopping distance reached.

What can we do?

Arnold, Vaman, Szepietwoski, Wong, 1212.3321

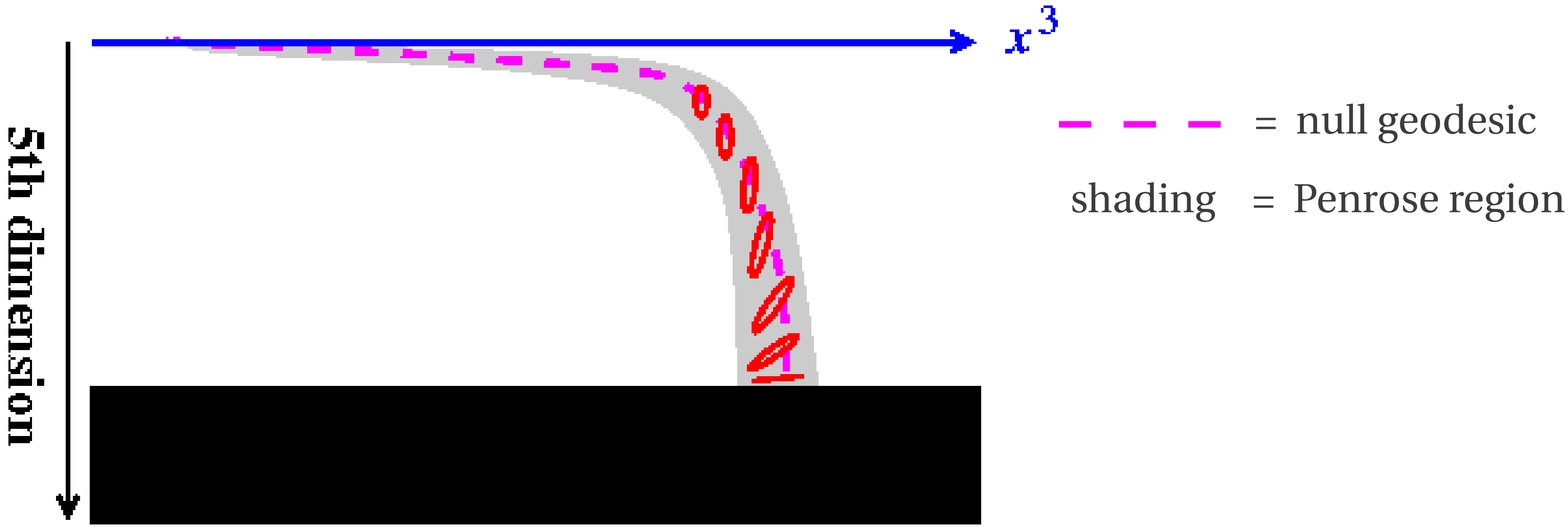


Answer: Replace the high-energy graviton by a closed string quantized in the AdS-Schwarzschild background.

Problem: No practical calculation method for quantizing strings in generic backgrounds.

Solution: For our problem, it's good enough to replace AdS-Schwarzschild by a Penrose limit (with caveats to be mentioned later).

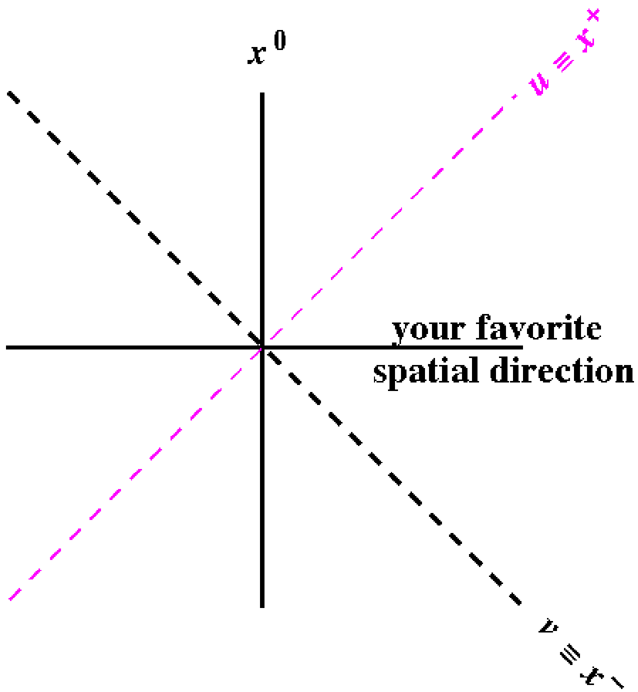
The Penrose Limit



In neighborhood of a null geodesic, can approximate the metric in a form (a pp-wave metric) for which practical calculations involving string quantization are possible!

How do we know if string will stay close enough to reference geodesic? Assume it does, calculate answer, and then check.

Reminder of string quantization in flat space



$$ds^2 = -du\,dv + d\vec{x}_\perp^{\,2}$$

Take world sheet time to be $\tau = u$

→ string decomposes into independent, transverse, harmonic oscillators

$$X_\perp^k(\sigma,\tau) = \sum_n X_{\perp n}^k(\tau)\, e^{in\sigma}$$

$X_{\perp n}^k(\tau) =$ harm. osc. w/ frequency $\omega_{k,n} = \dfrac{n^2}{(\alpha' p^u)^2}$

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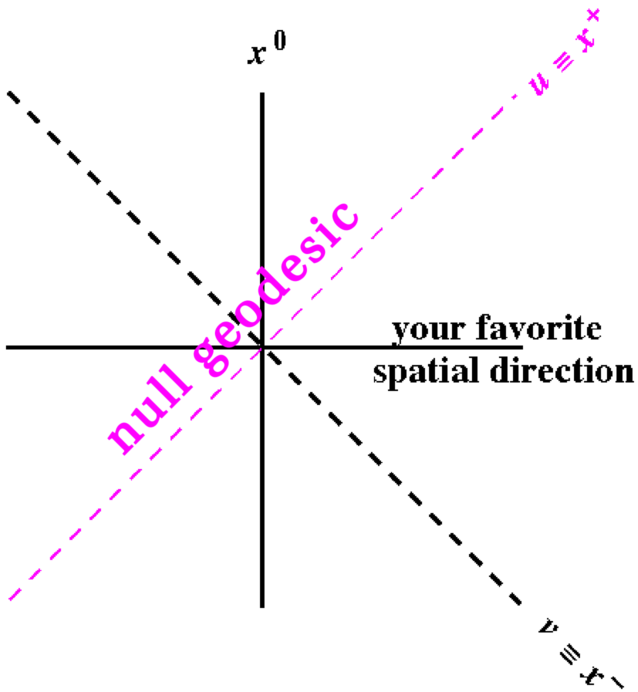

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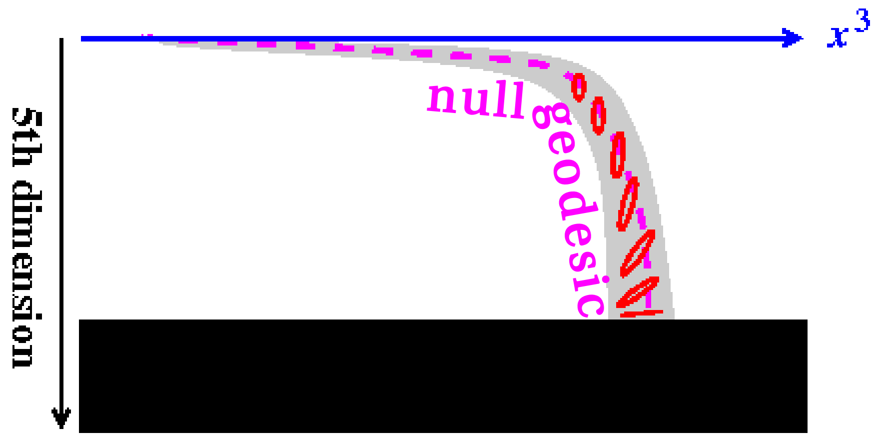
String quantization in Penrose limit



Take u = affine parameter along reference null geodesic

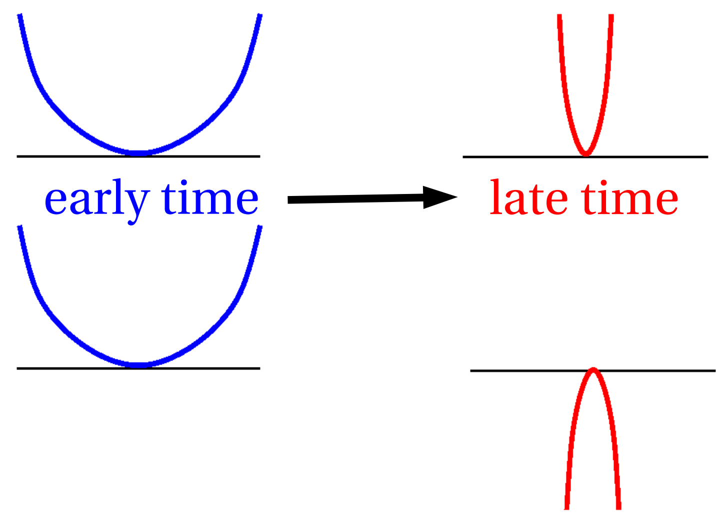
$$ds^2 = -du\,dv + \cdots$$

Take world sheet time to be $\tau = u$

$\rightarrow$ as before, but HO's pick up tidal force terms from curvature of space-time:

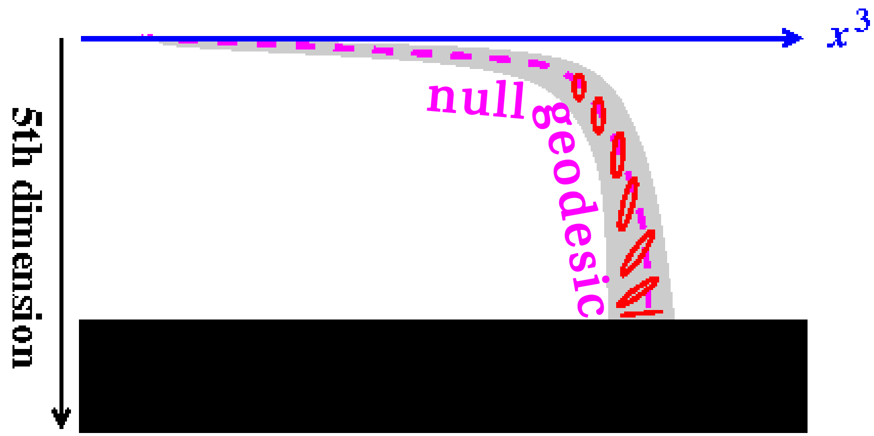
$$\omega_{1,n}^2 = \omega_{2,n}^2 = \frac{n^2}{(\alpha' p^u)^2} + \frac{1}{2}G(\tau)$$

$$\omega_{3,n}^2 = \frac{n^2}{(\alpha' p^u)^2} - G(\tau)$$



$G(\tau)$ grows as one moves away from the boundary.

String quantization in Penrose limit



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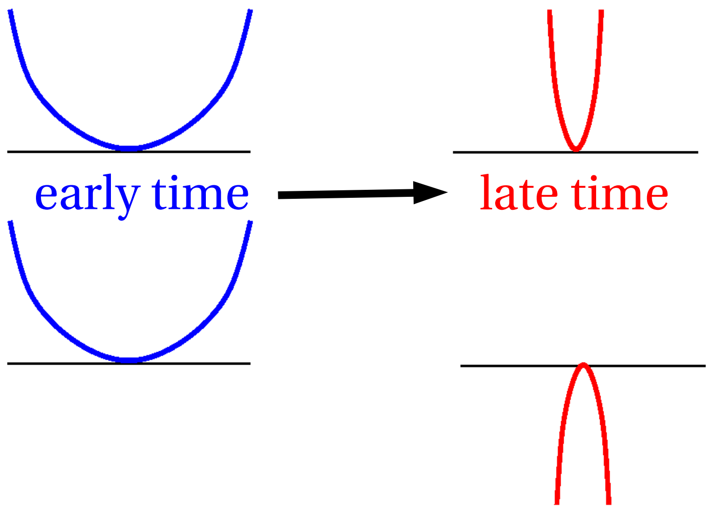
STRING TENSION

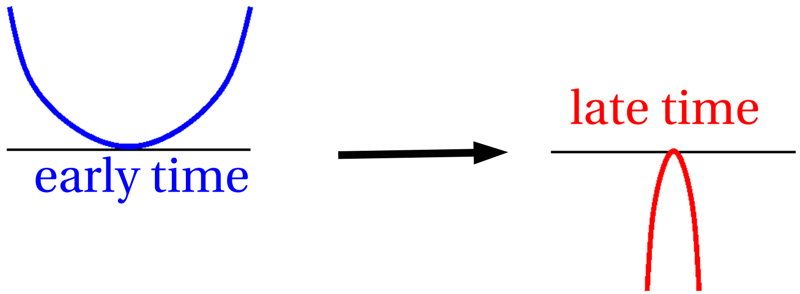
TIDAL FORCES



$G(\tau)$ grows as one moves away from the boundary.

Now just a QM problem:

Need QM solution to a time-dependent harmonic oscillator that starts in its ground state.

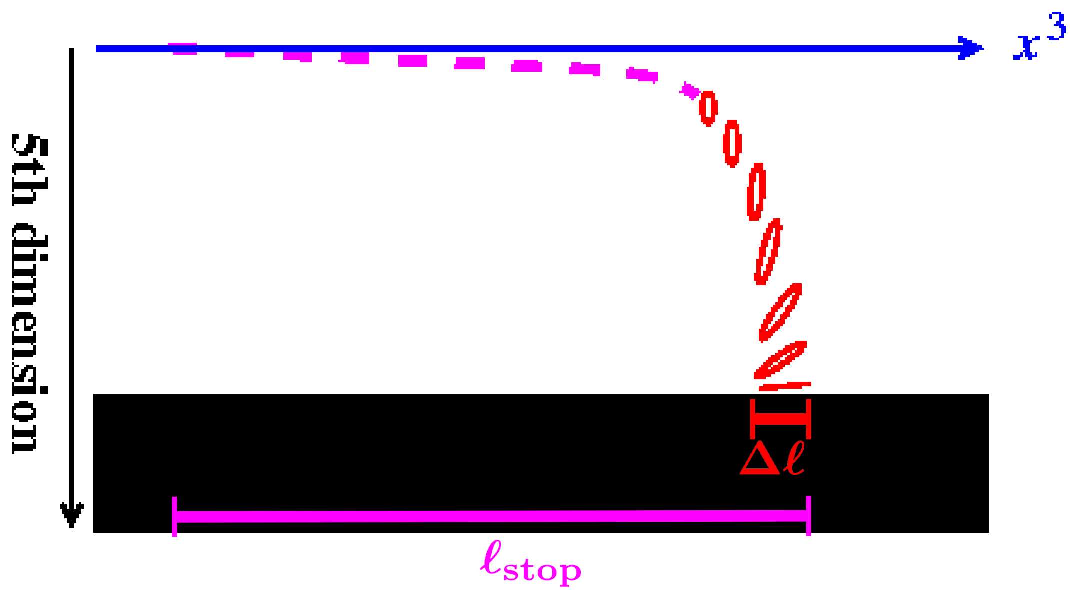


At late times, dynamics become classical.

Can calculate late-time probability distribution for each oscillator (i.e. for the amplitude X^k_n of each string harmonic).

$\rightarrow$ Can calculate the late-time size of the classical string.

Results



For case $\ell_{\rm stop} \lesssim \lambda^{-1/6}\ell_{\rm max}$ where string excitation is important,

$$(\Delta\ell)_{\rm rms} \simeq 0.8660\, \lambda^{-1/4} \ell_{\rm stop} \ln^{1/2} n_*$$

where

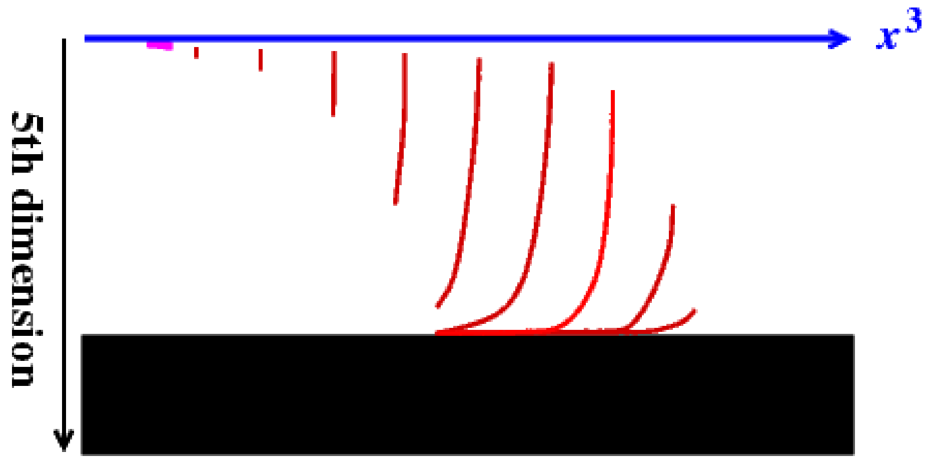
$$n_* \sim \text{\# harmonics excited} \sim \frac{\lambda^{-1/6}\ell_{\rm max}}{\ell_{\rm stop}}$$

Moral: Stretching of string has negligible impact on jet stopping unless n_* is exponentially large !

And what if it is? ...

Large $\ln(n_*)$

$$\lambda^{-1/4}\ln^{1/2} n_* \sim 1$$



Note: Penrose limit breaks down

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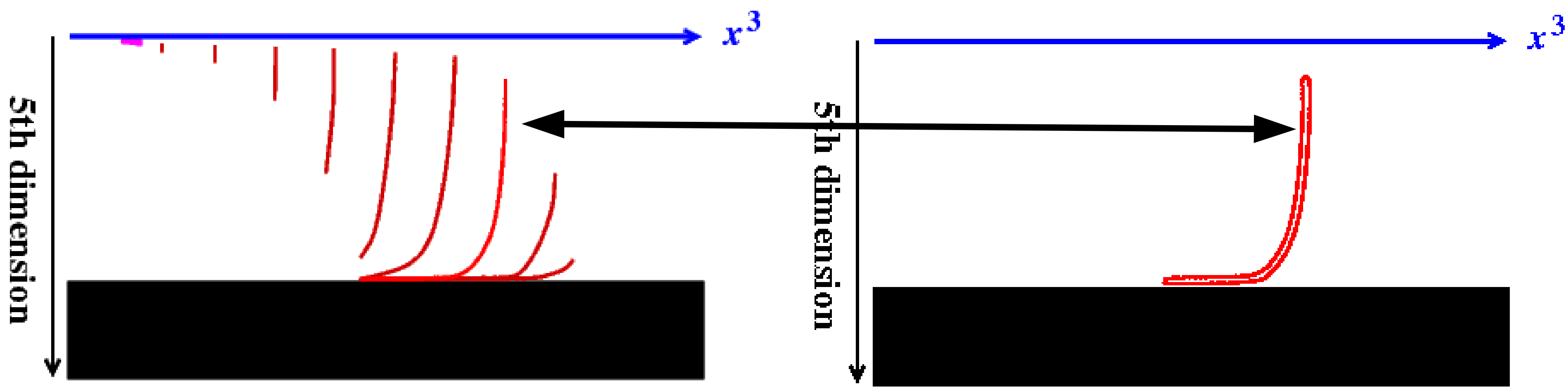
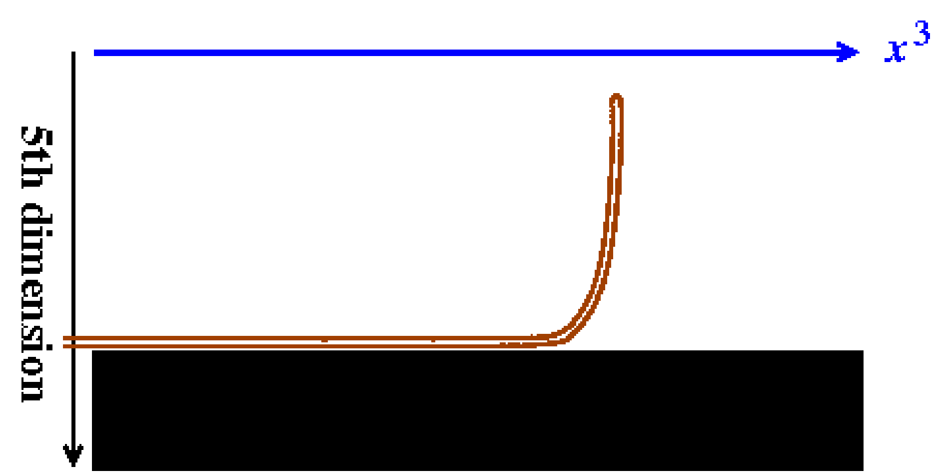


Note: Penrose limit breaks down

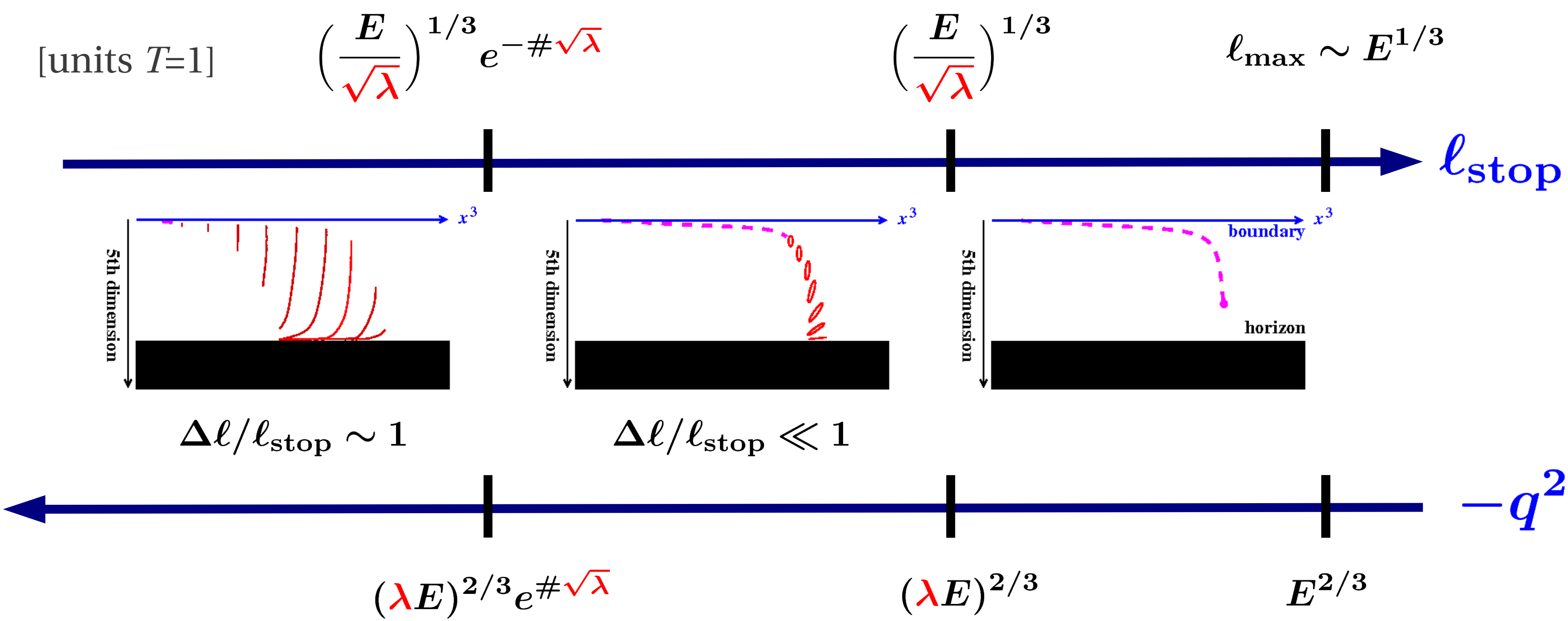
qualitatively similar to



Gubser, Gulotta, Pufu, Rocha (2008)

Summary

At very high energy $E \gg \sqrt{\lambda}\, e^{\#\sqrt{\lambda}} T$



What happens as λ decreases?

All the above scales coalesce as $\lambda \to 1$.